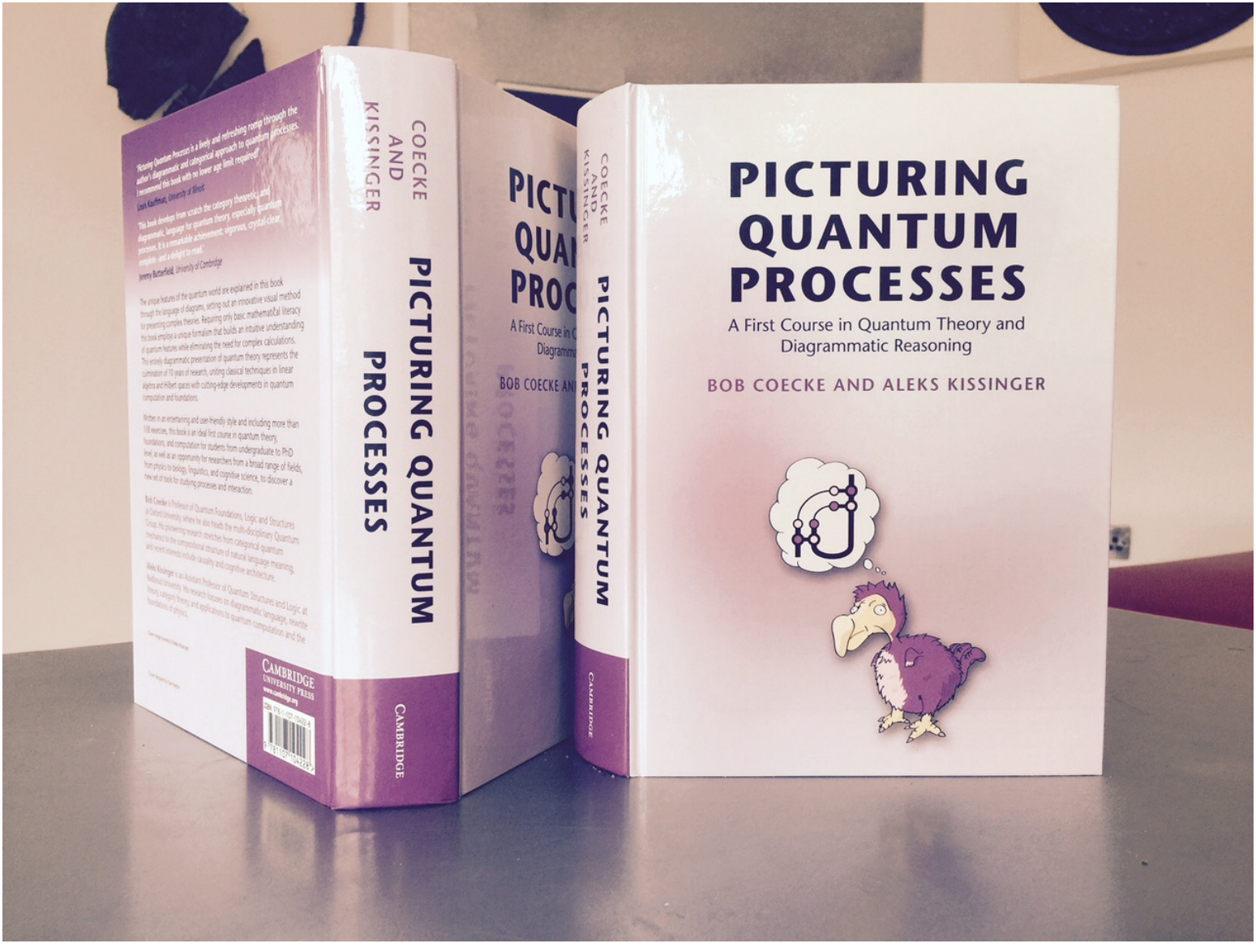

PICTURING QUANTUM PROCESSES
A First Course in Quantum Theory and Diagrammatic Reasoning
BOB COECKE AND ALEKS KISSINGER
COECKE AND KISSINGER
CAMBRIDGE

Can QM be formulated in pictures ?

B. Coecke (2005) Kindergarten quantum mechanics. quant-ph/0510032

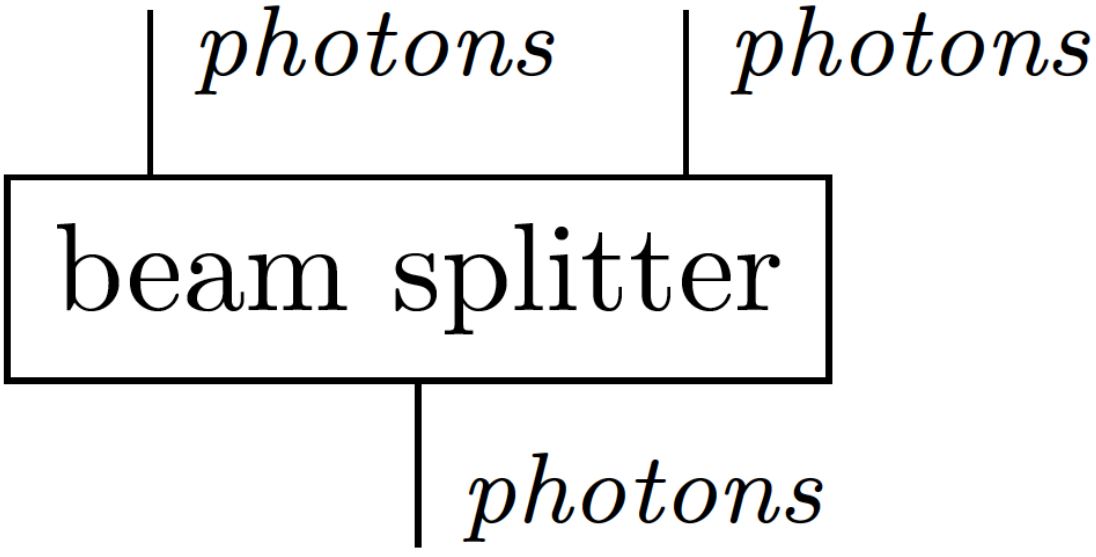
YES !

YES !

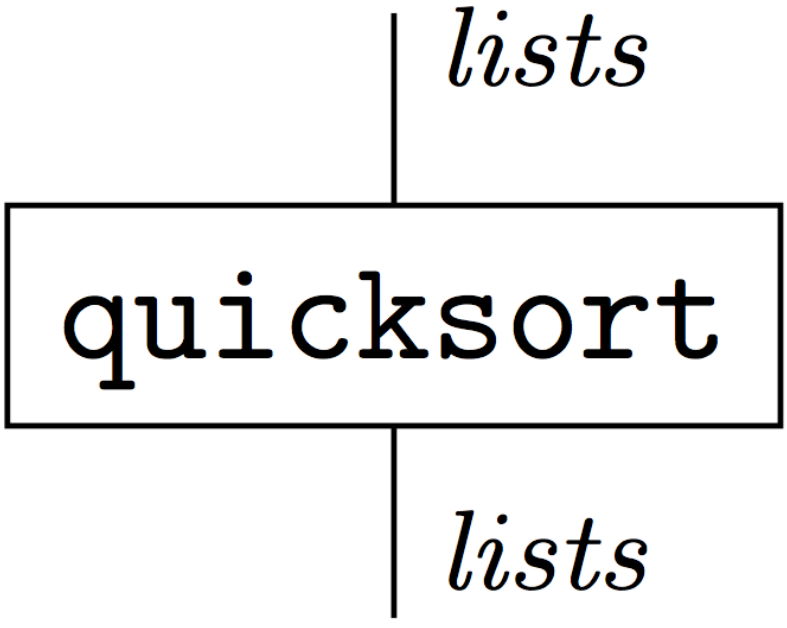
— Ch. 1 – Processes as diagrams —

The art of progress is to preserve order amid change, and to preserve change amid order.

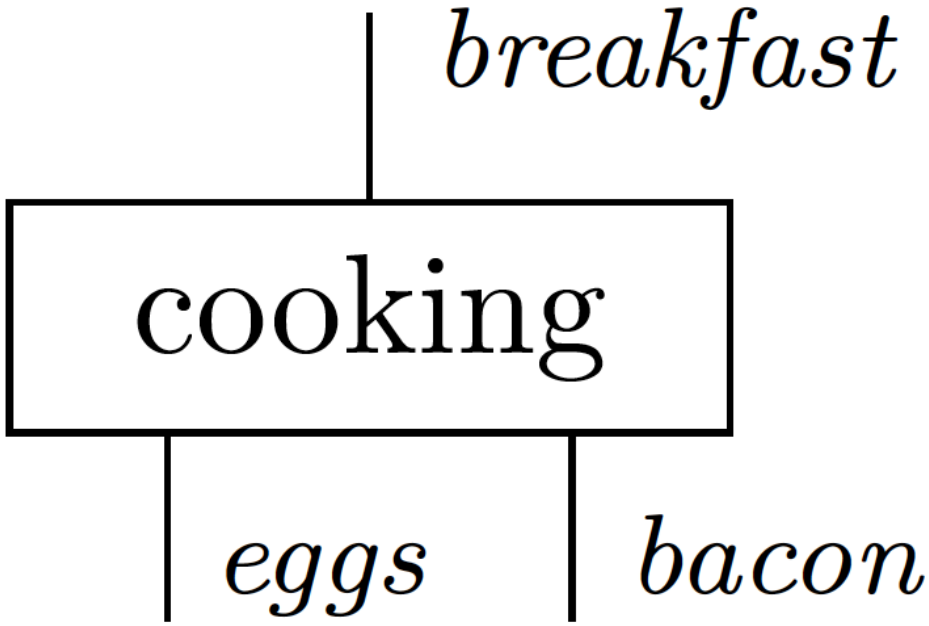
— Alfred North Whitehead, Process and Reality, 1929.


photons
photons
beam splitter
photons

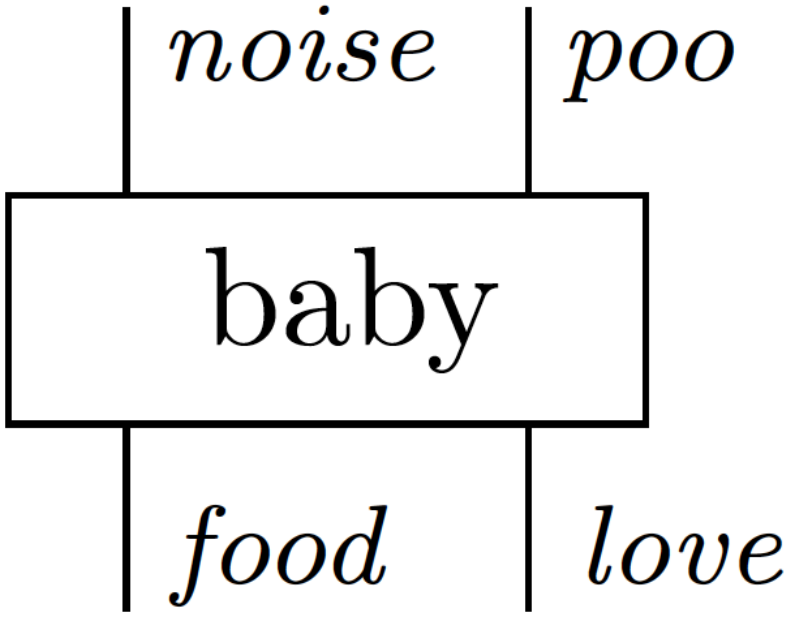
– processes as boxes and systems as wires –



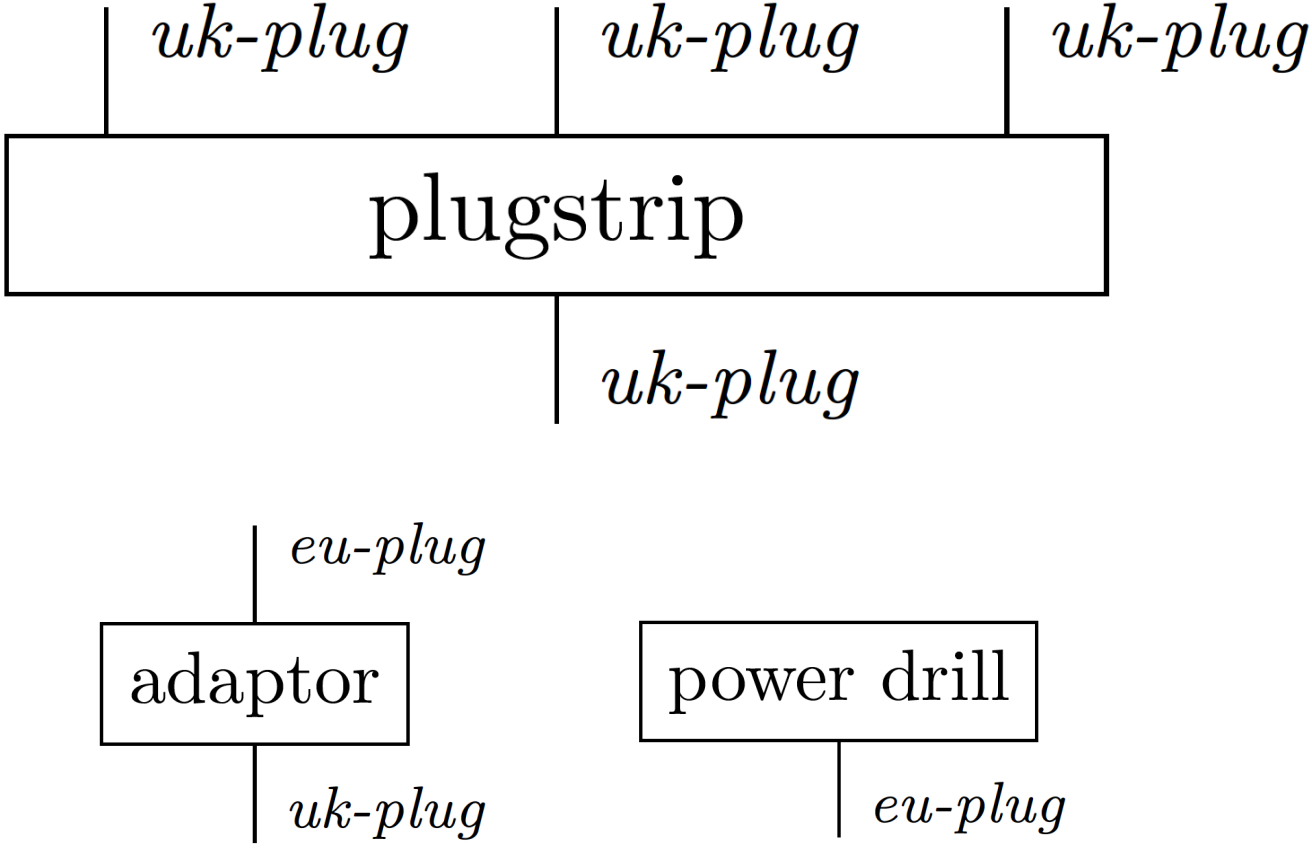
– processes as boxes and systems as wires –



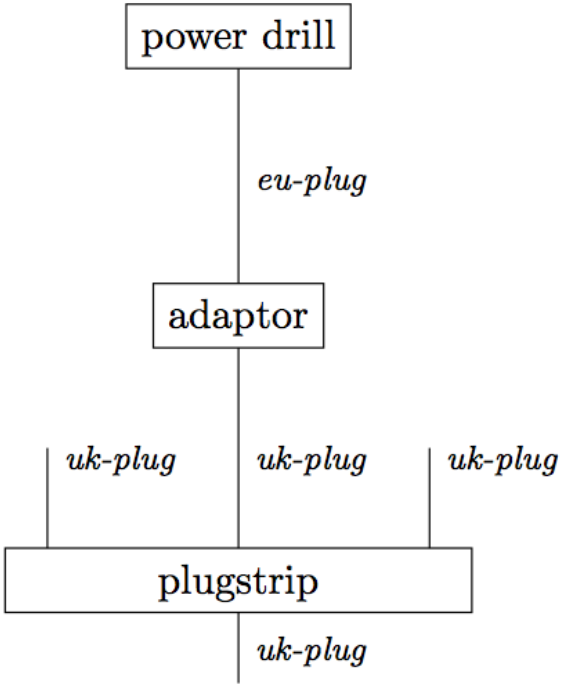
– processes as boxes and systems as wires –



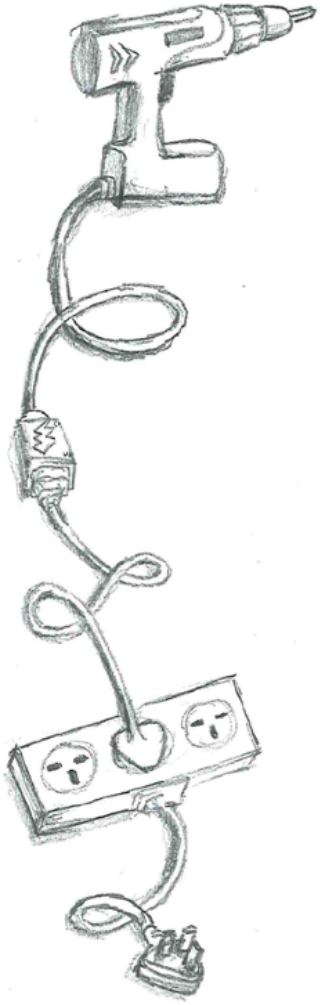
– composing processes –



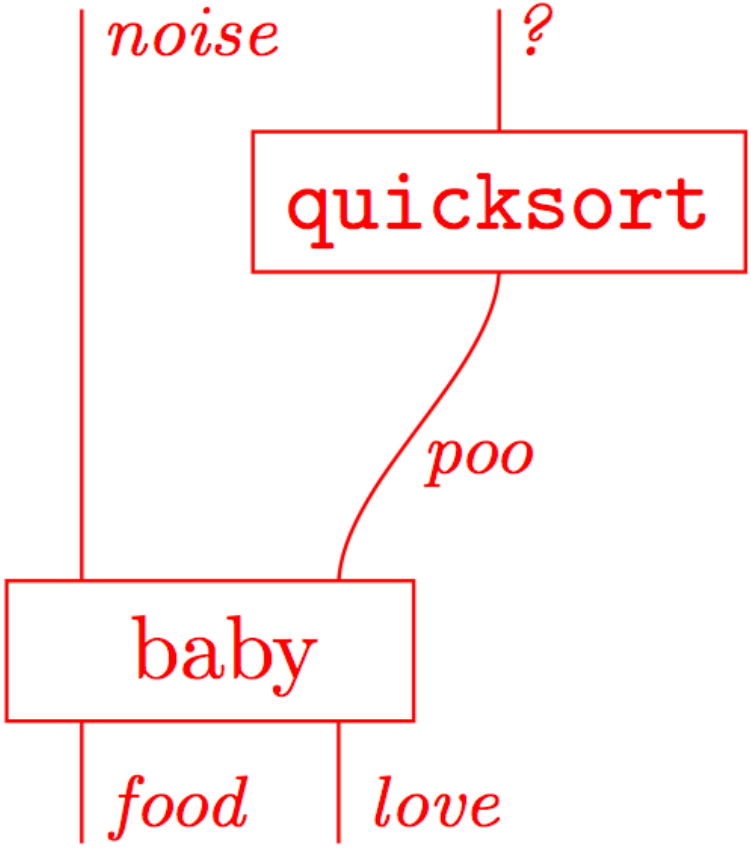
– composing processes –



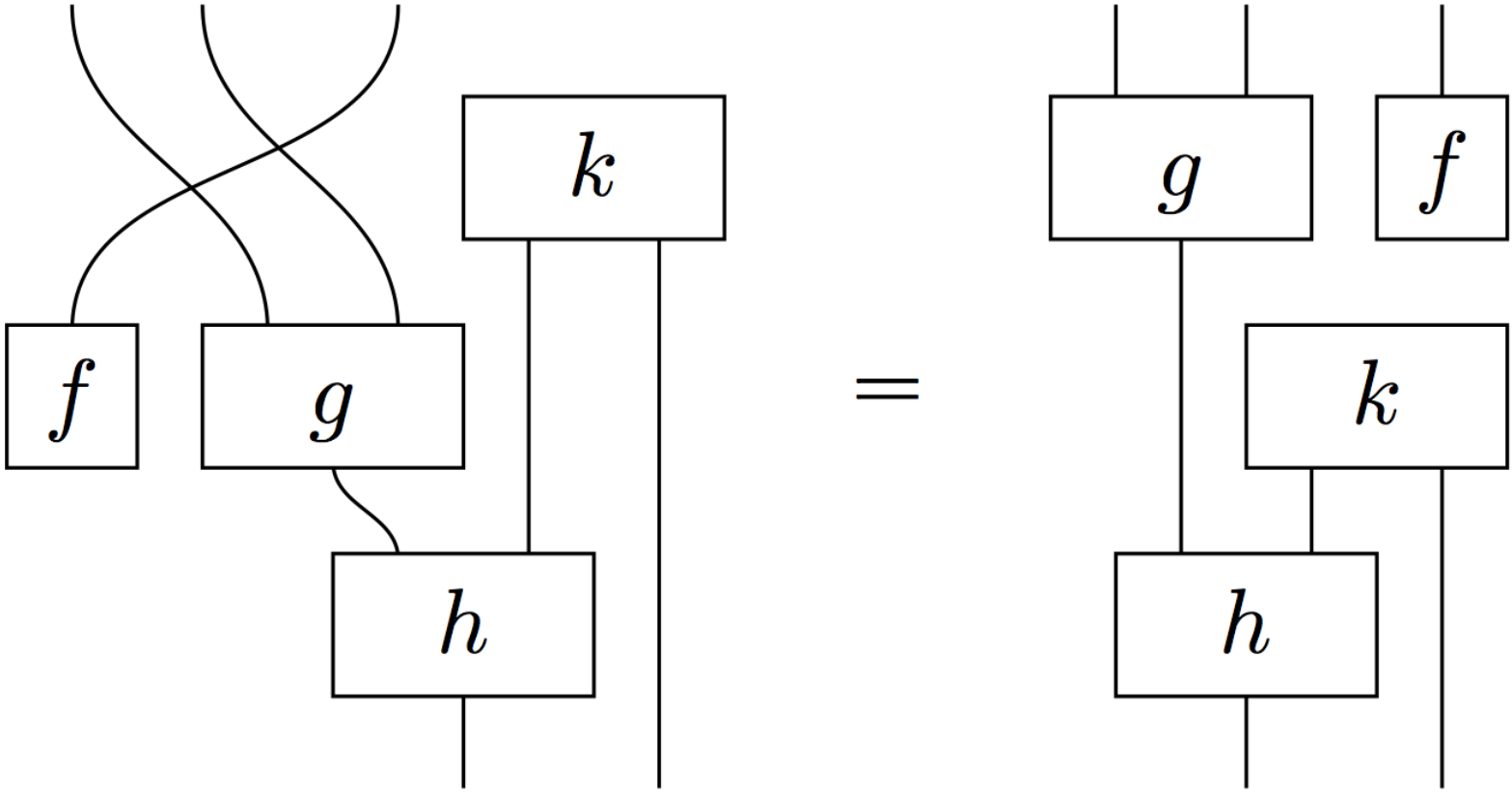
:=



– composing processes –

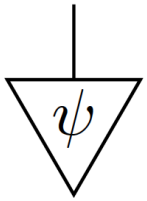


– tautologies –

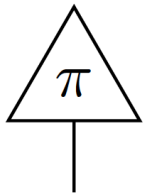


– *special processes/diagrams* –

State :=

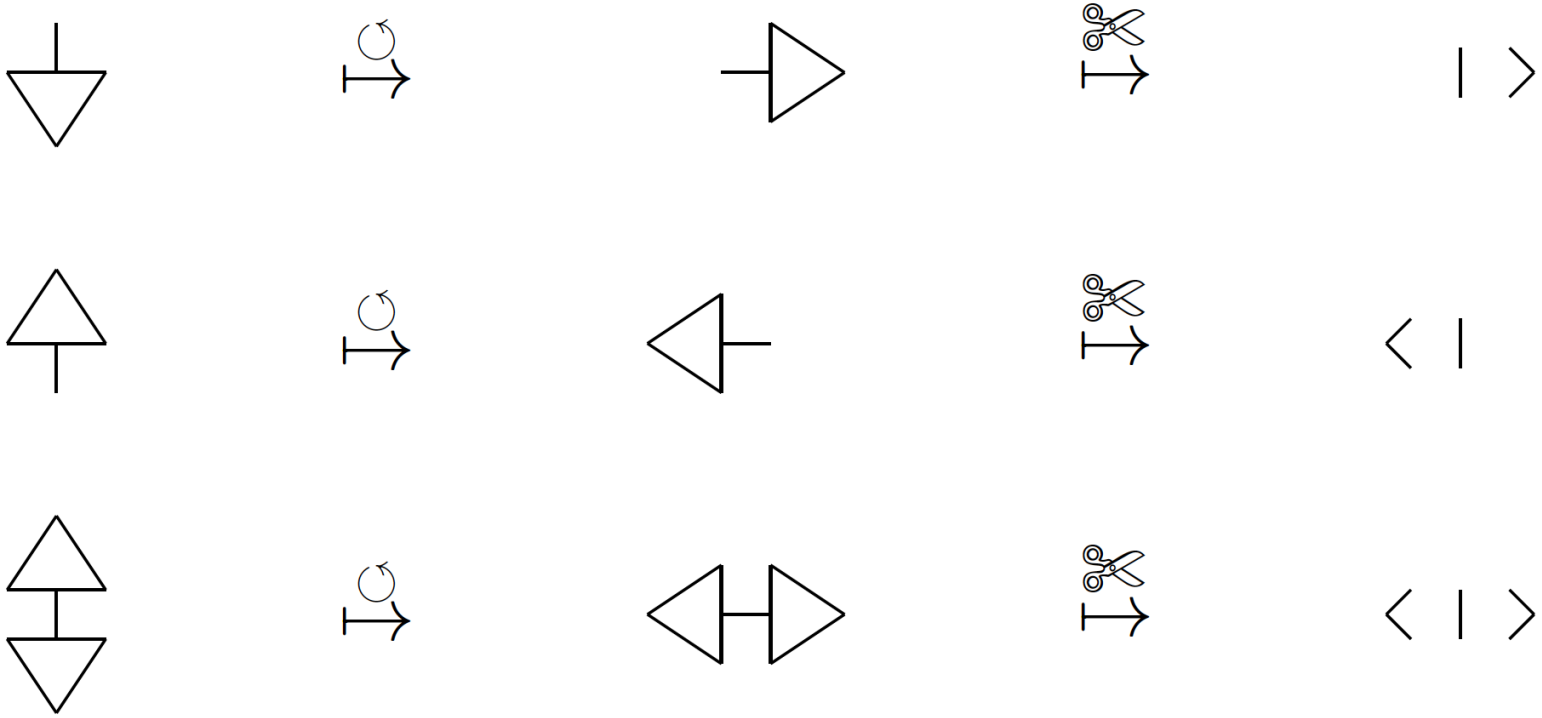


Effect / Test :=

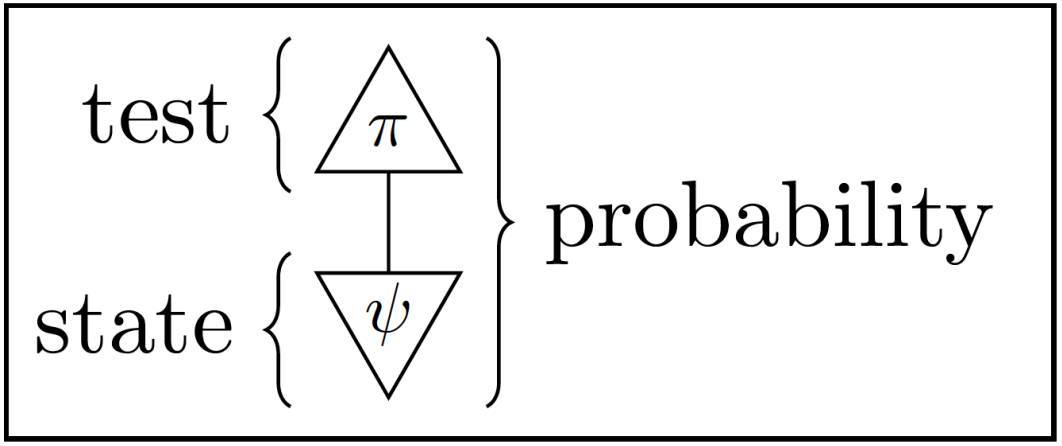


Number :=

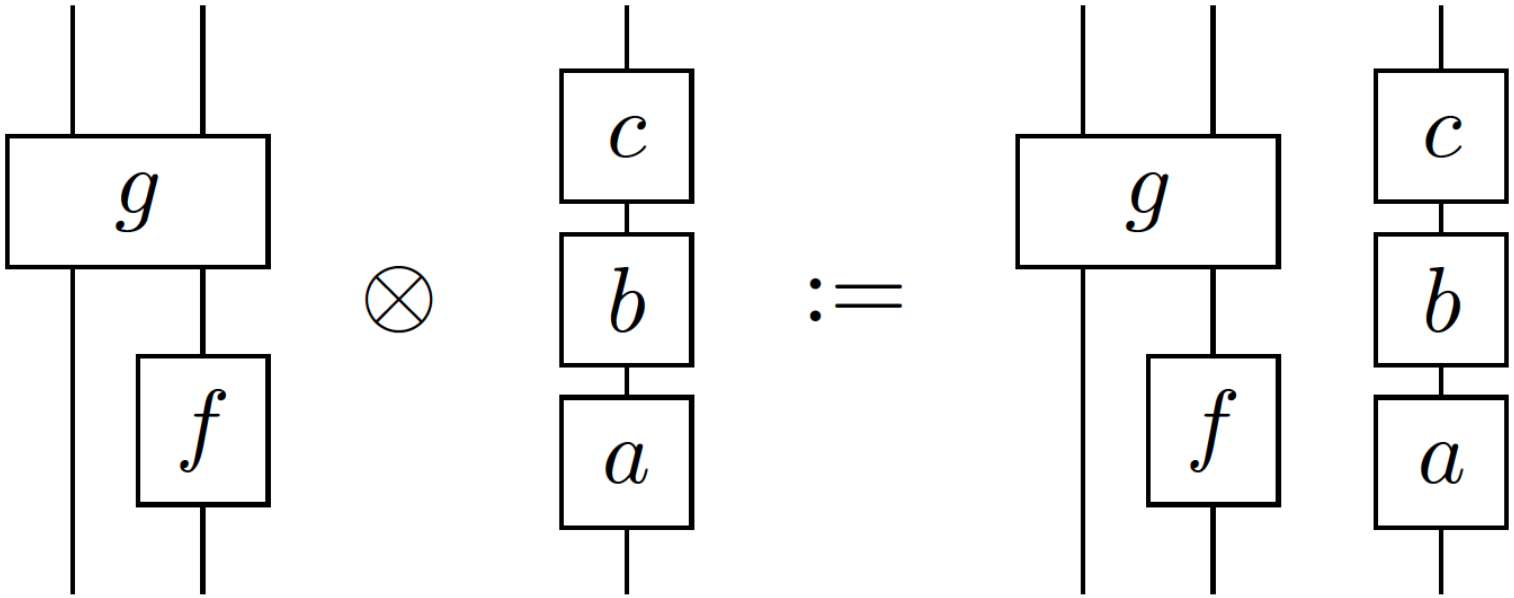
– special processes/diagrams –



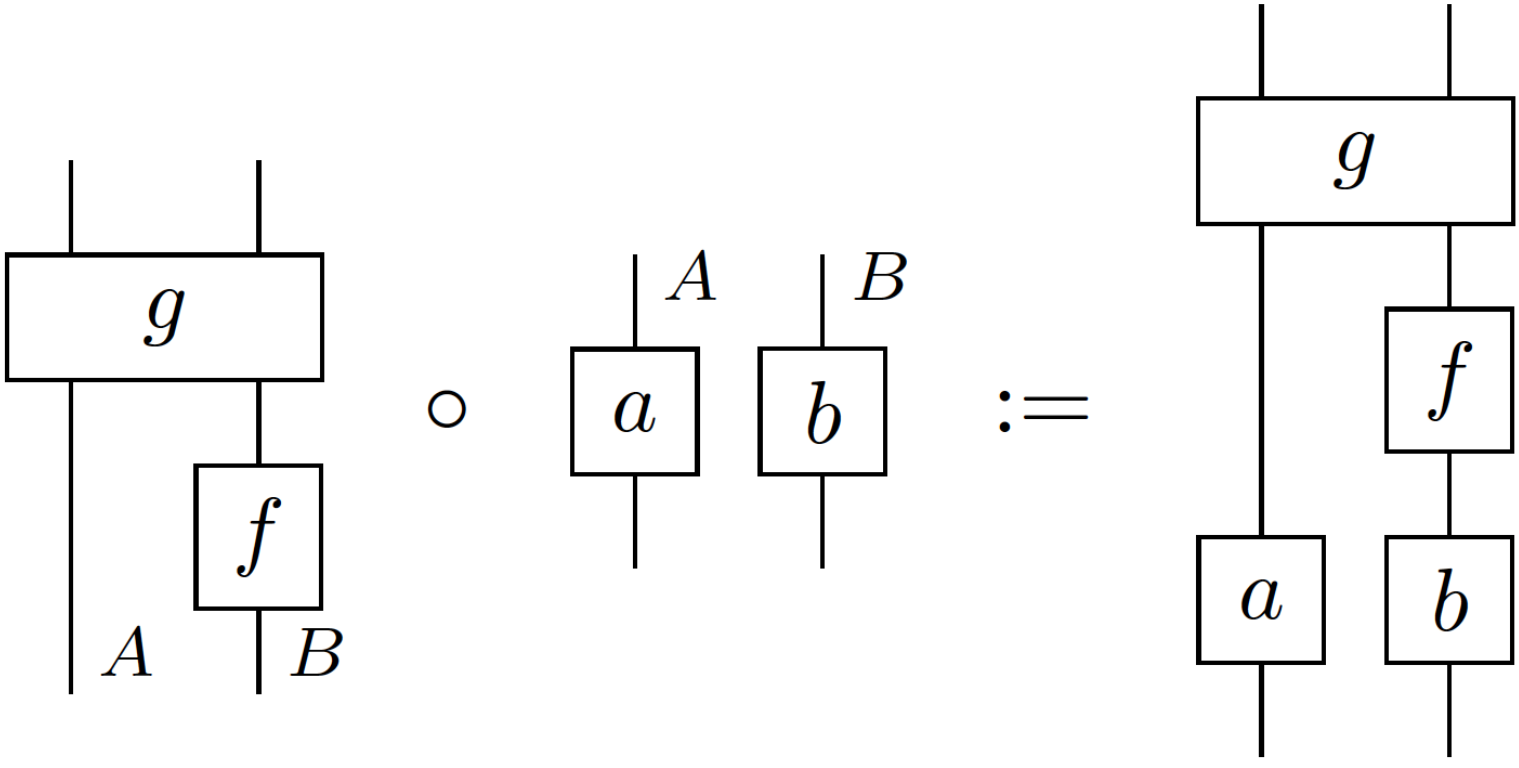
Born rule :=



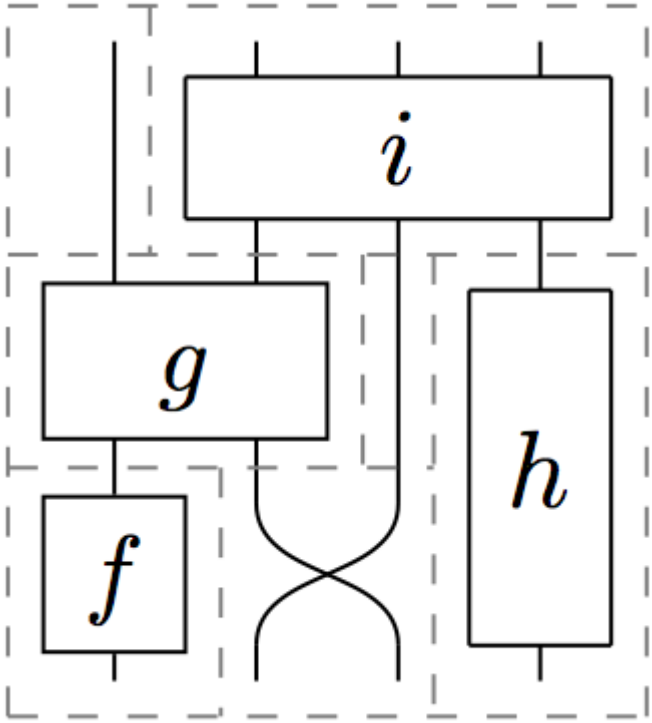
– “$f \otimes g$” := “f **while** g ” –



– “$f \circ g$” := “f **after** g” –



Defn. ... := can be build with $\otimes$ and $\circ$.



– circuits –

Defn. ... := can be build with $\otimes$ and $\circ$.

Fact. ...are boring.

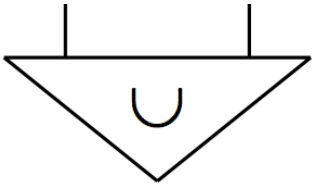
— Ch. 2 – String diagrams —

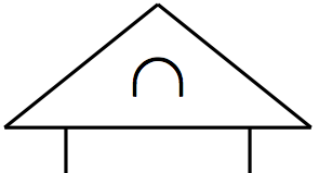
When two systems, of which we know the states by their respective representatives, enter into temporary physical interaction due to known forces between them, and when after a time of mutual influence the systems separate again, then they can no longer be described in the same way as before, viz. by endowing each of them with a representative of its own. ***I would <u>not</u> call that <u>one</u> <u>but</u> rather <u>the</u> characteristic trait of quantum mechanics****, the one that enforces its entire departure from classical lines of thought.*

— Erwin Schrödinger, 1935.

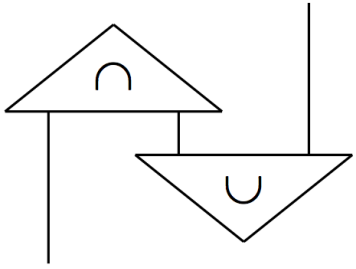
– *TFAE* –

1. ‘Circuits’ with cup-state and cup-effect:

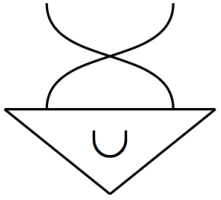




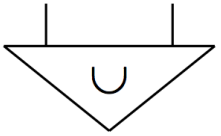
which satisfy:

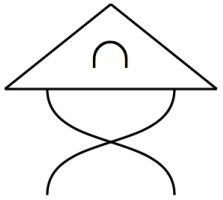


=

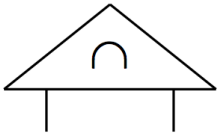


=

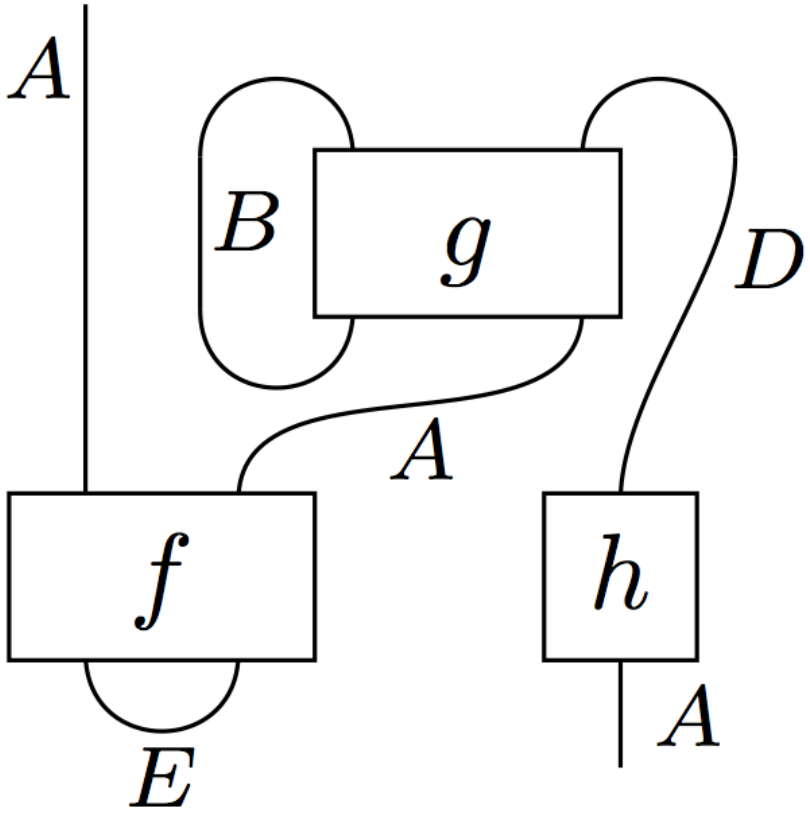




=

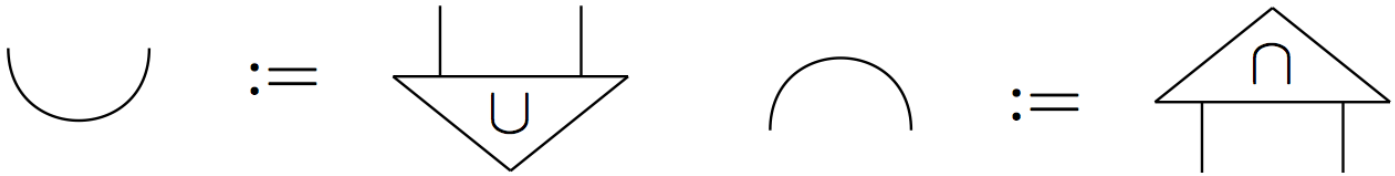


2. diagrams allowing in-in, out-out and out-in wiring:

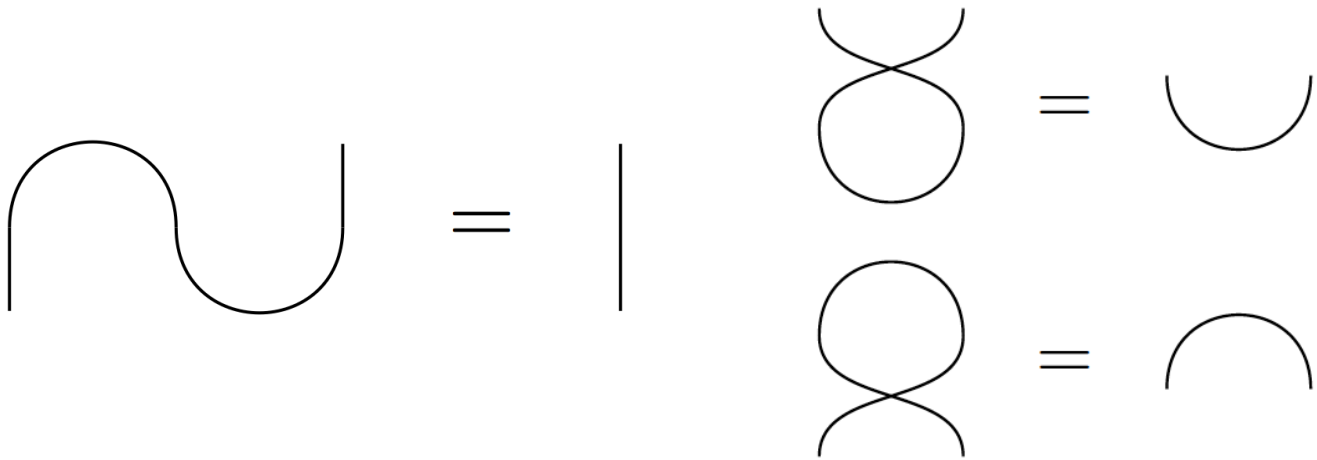


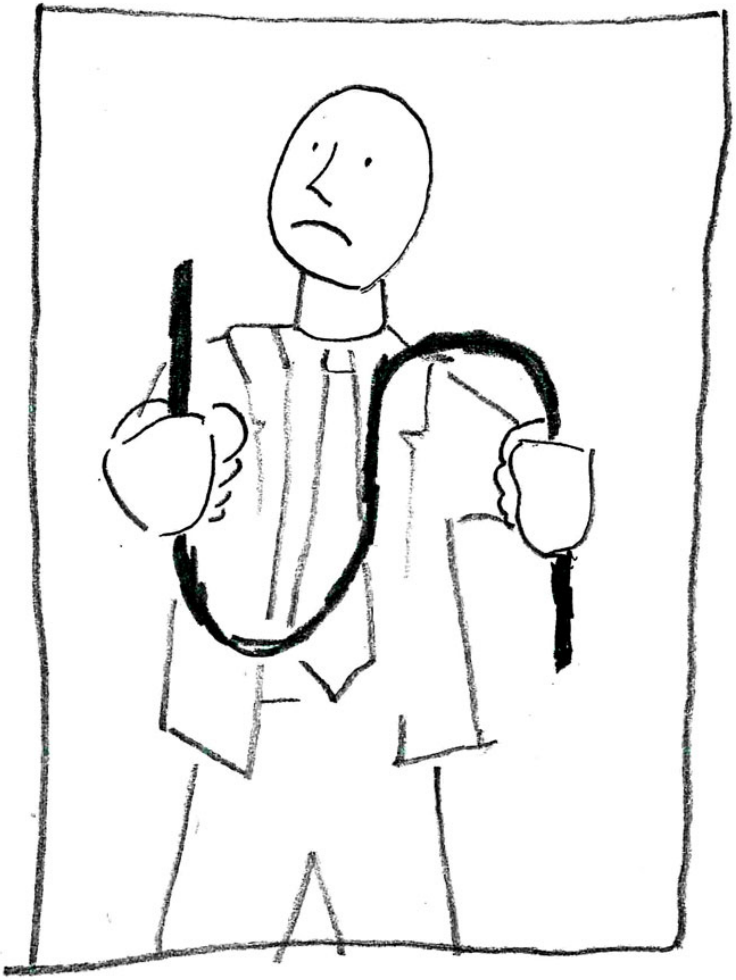
– *TFAE* –

From **1.** to **2.**:

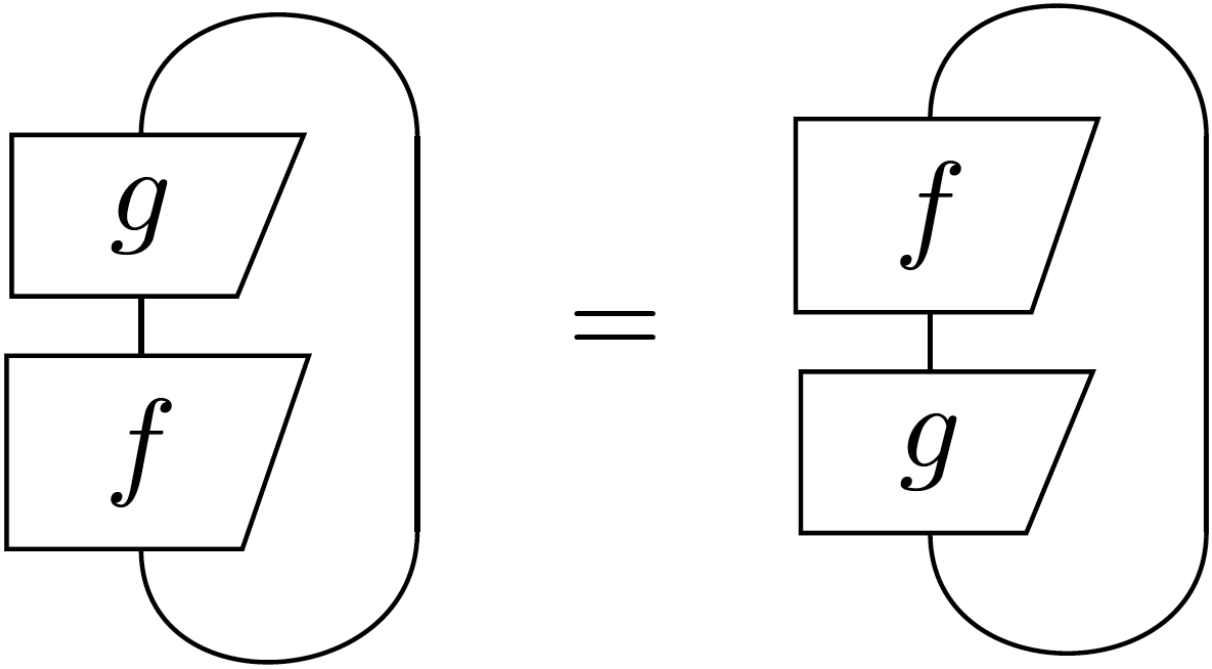


so that:

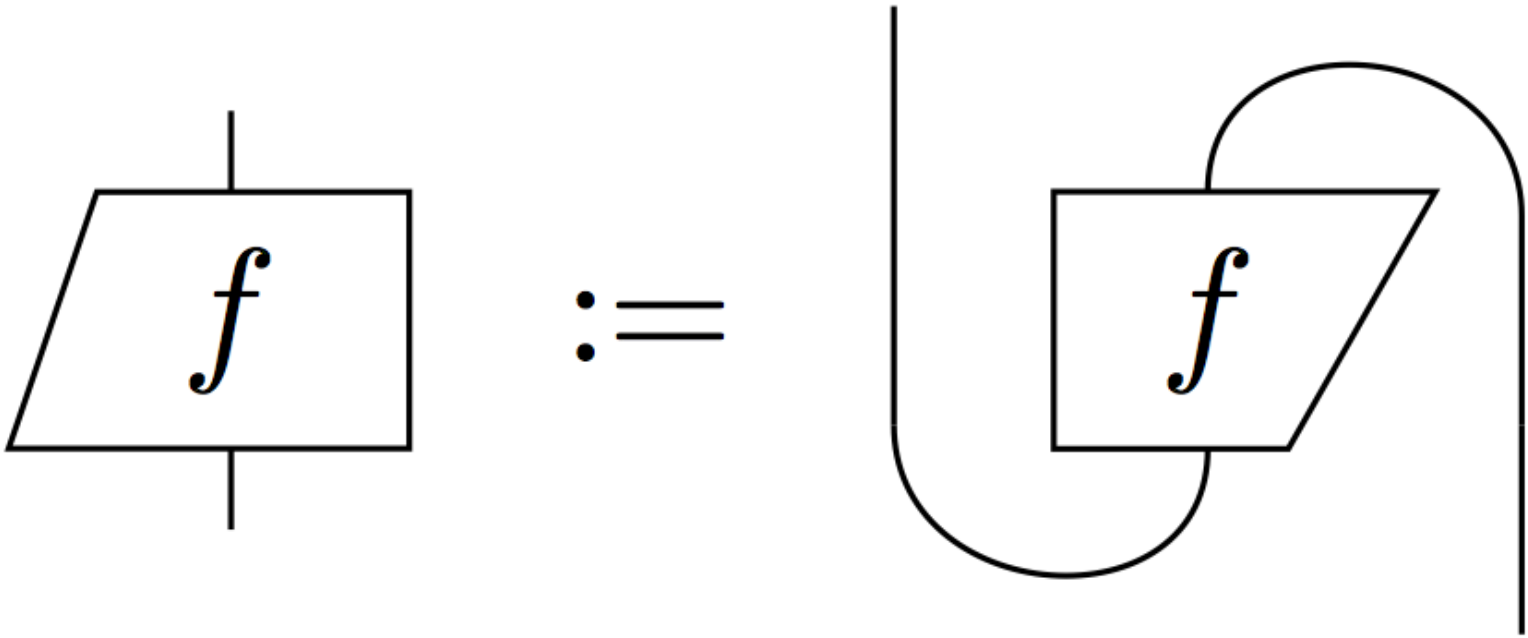


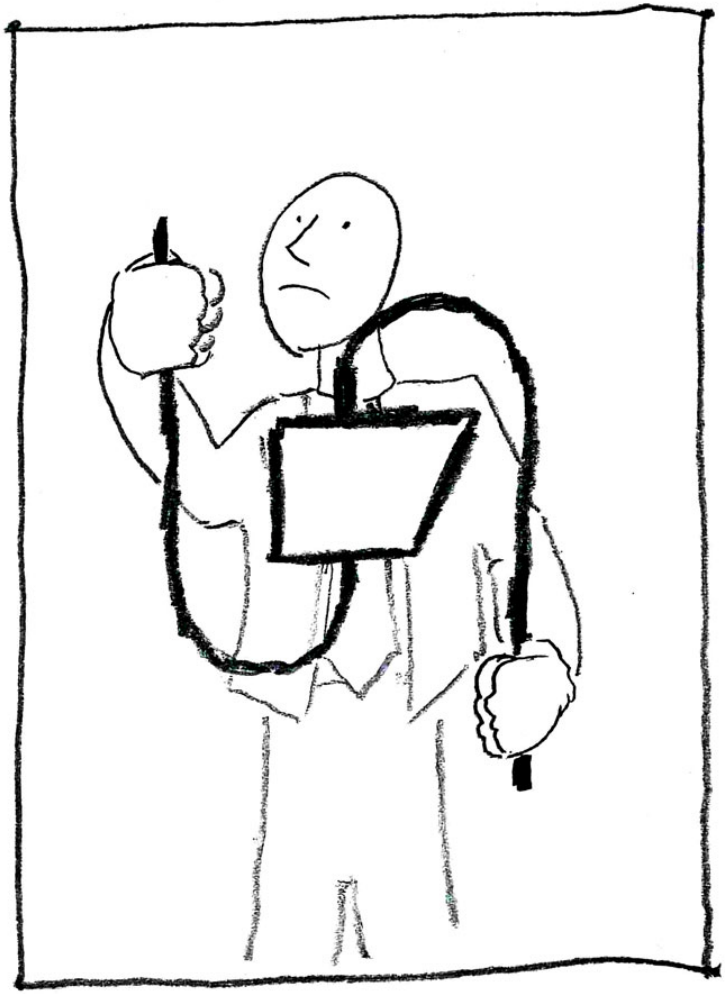


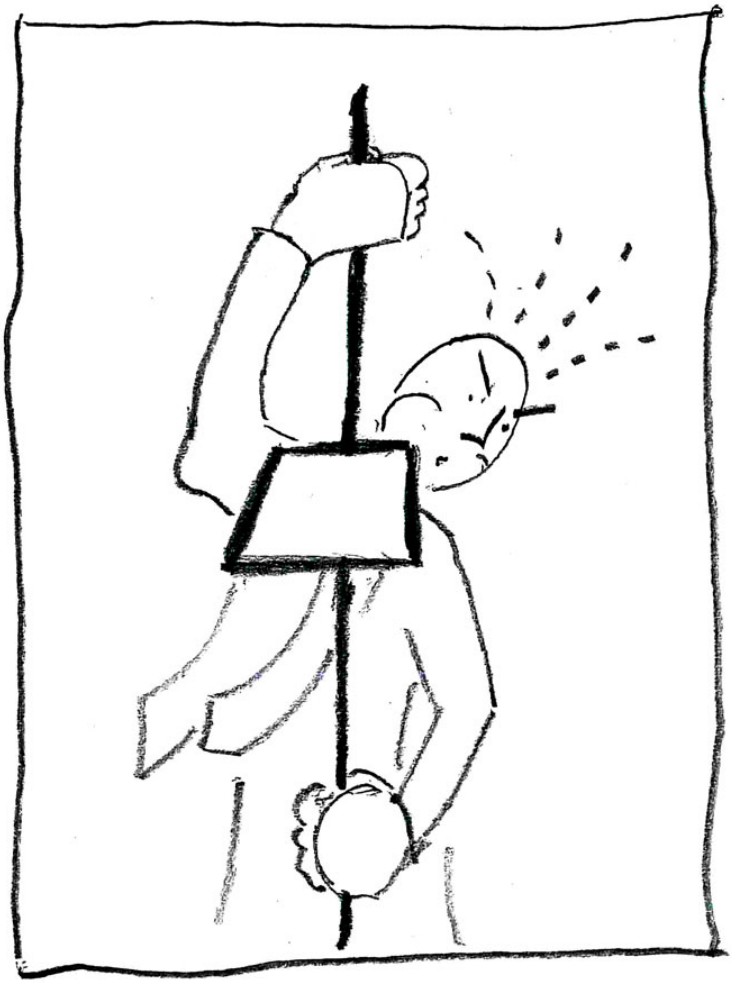
– tautology –



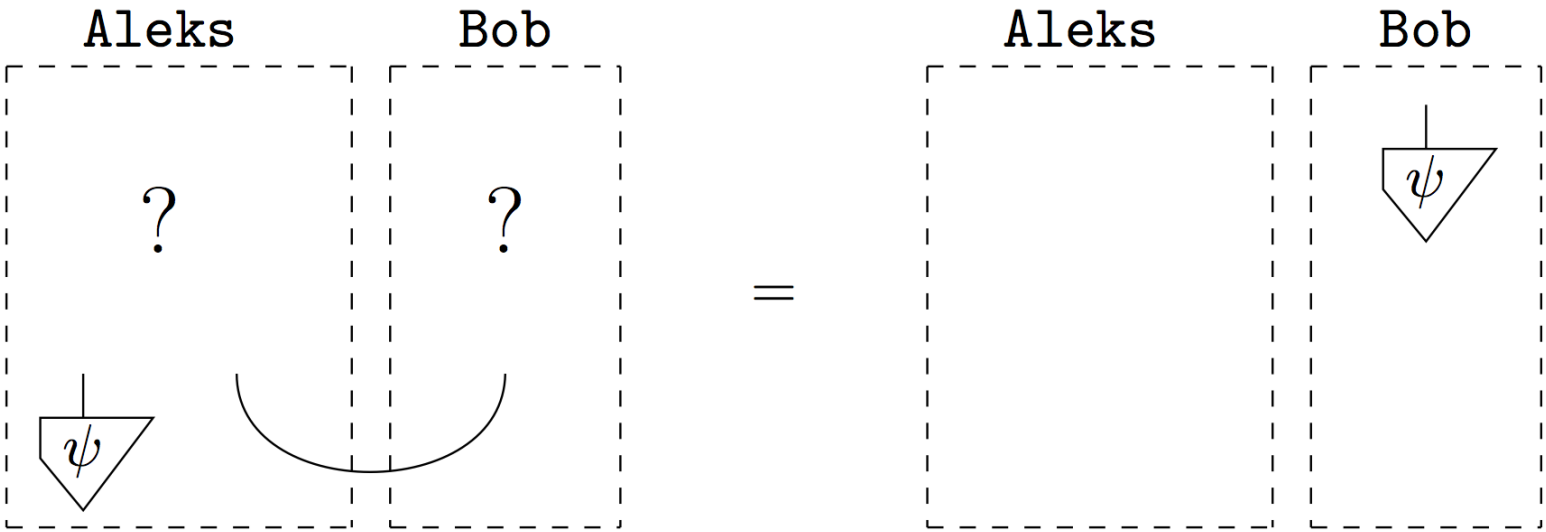
– implicit diagrammatic notion –



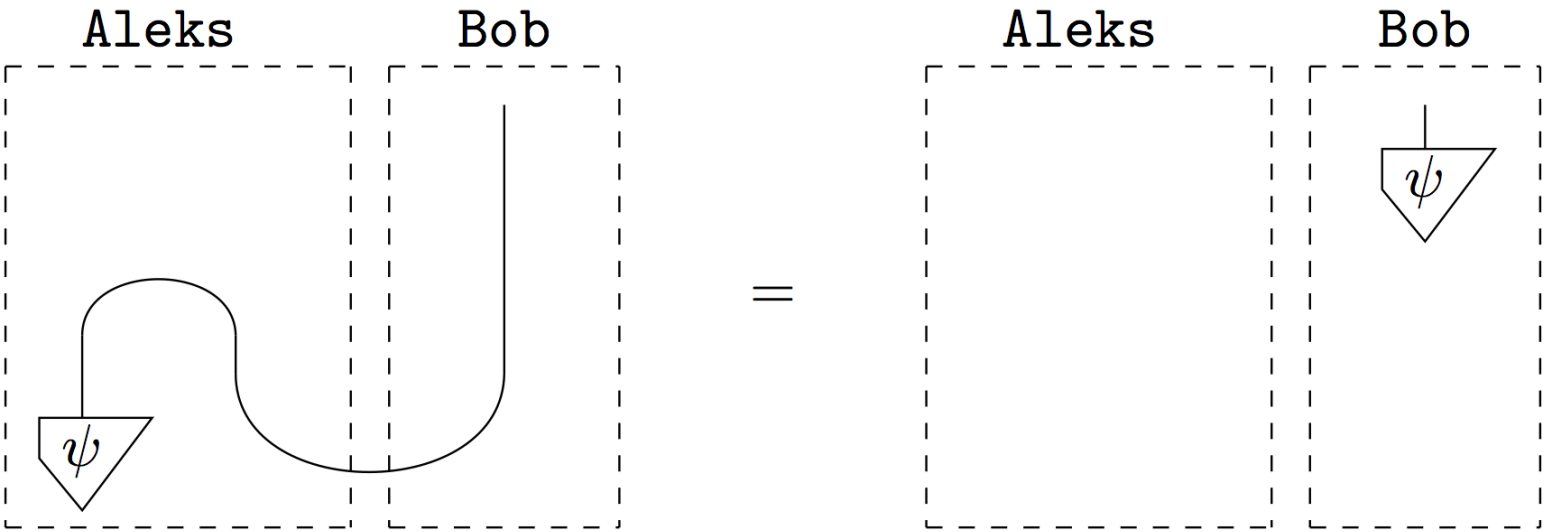




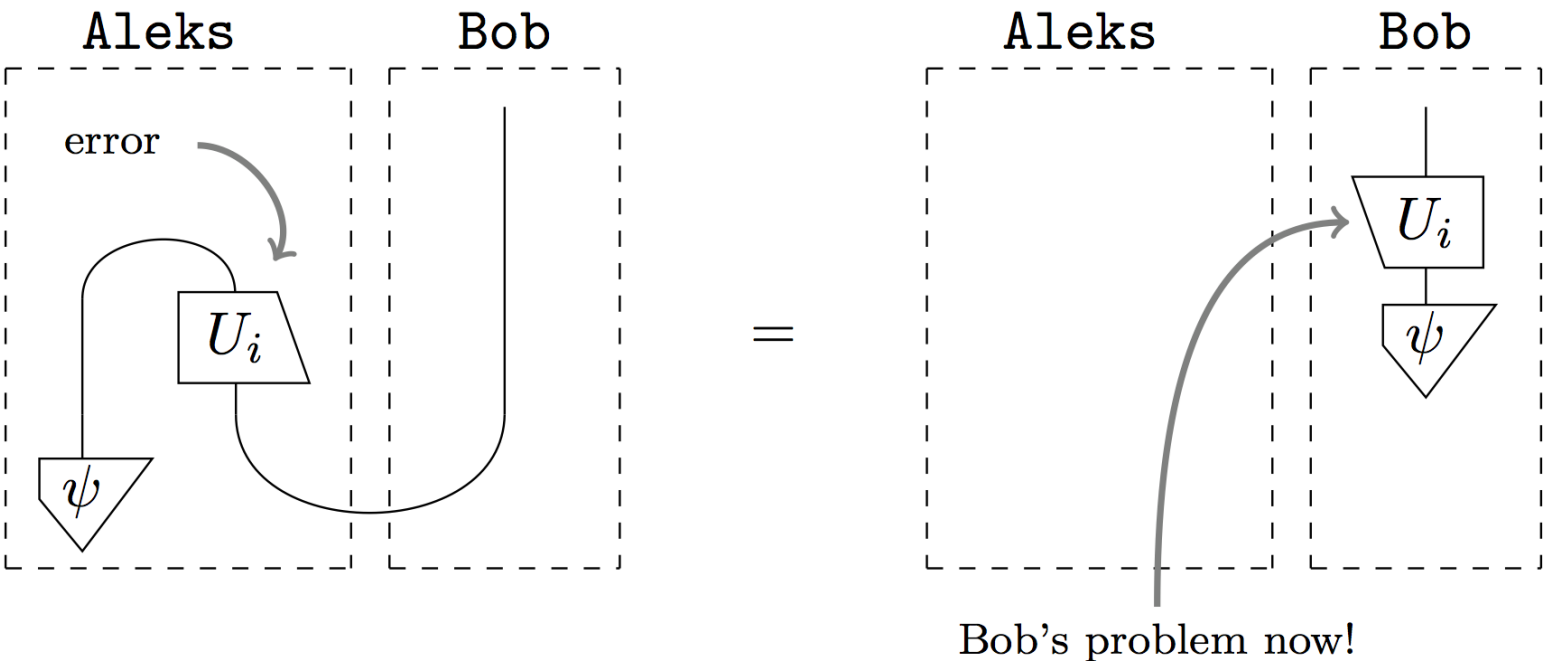
– quantum teleportation –



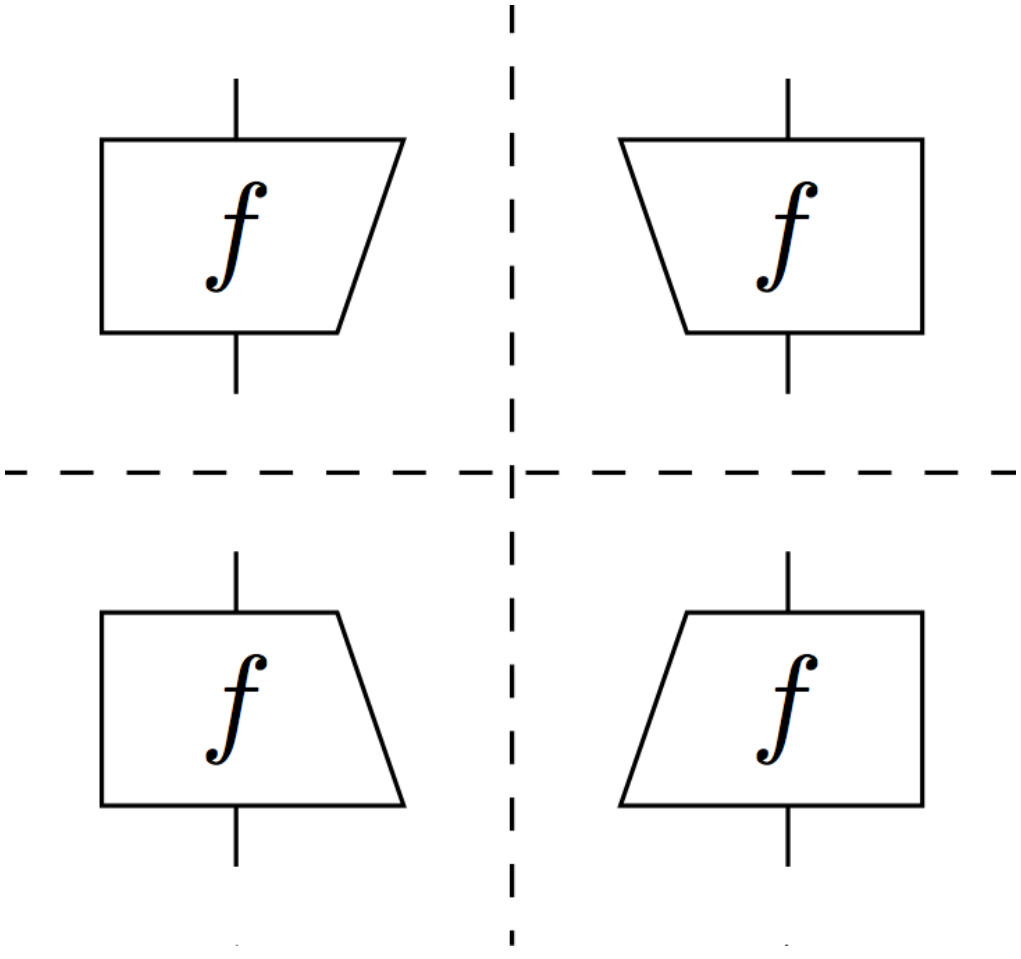
– quantum teleportation –



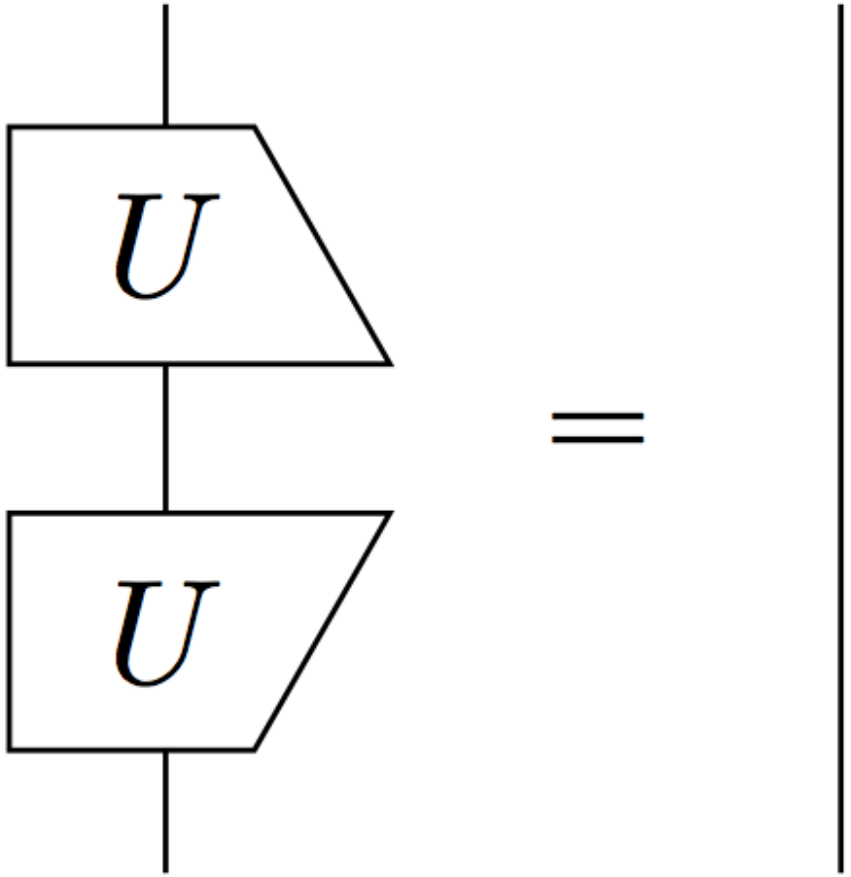
– quantum teleportation –



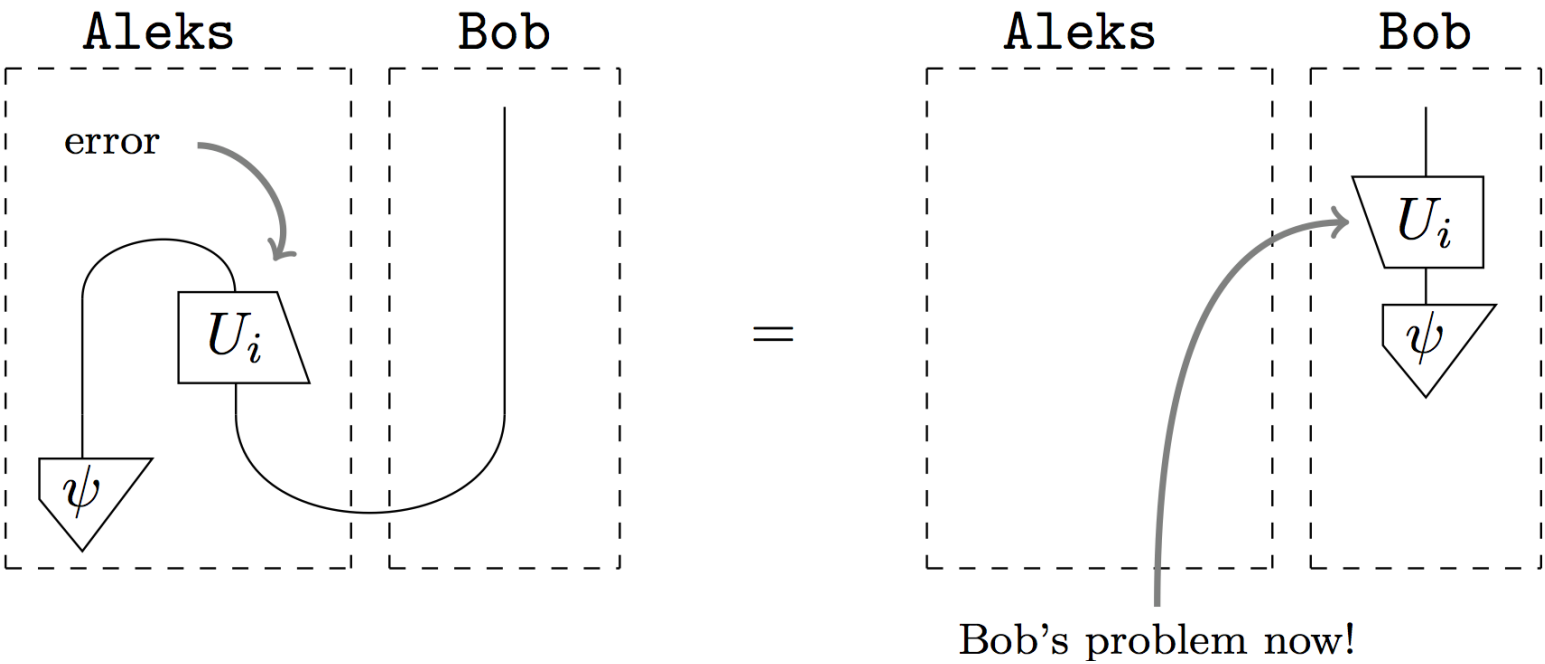
– adjoint & conjugate –



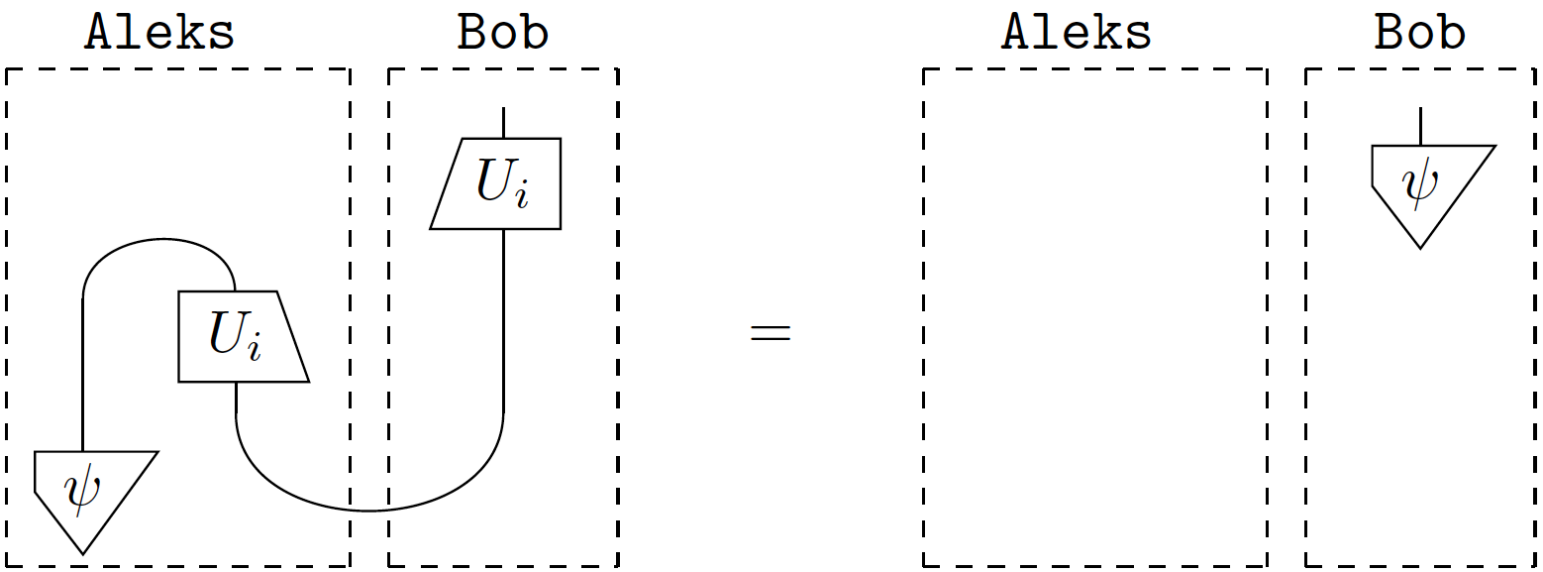
– isometry (& unitarity) –



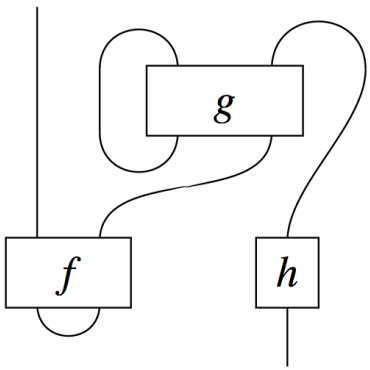
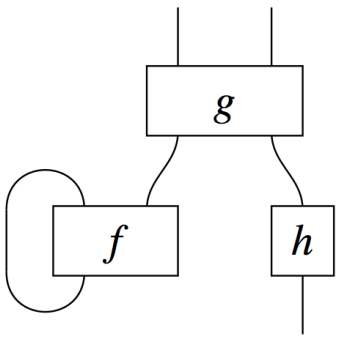
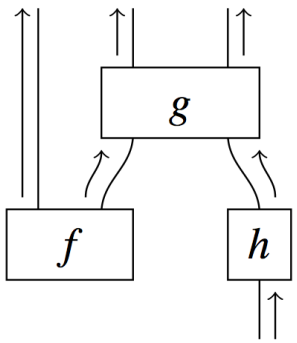
– quantum teleportation –



– quantum teleportation –



– SMCs as diagrams –

compact closed	traced	boring plain ones
		
string diagrams	**diagrams**	**circuits**
no ins/outs	outs to ins	causal structure

— Ch. 3 – Hilbert space from diagrams —

I would like to make a confession which may seem immoral: ***I do not believe absolutely in Hilbert space any more.***

— John von Neumann, letter to Garrett Birkhoff, 1935.

– *completeness* –

THM. (Selinger, 2008)

An equation between string diagrams holds, if and only if it holds for Hilbert spaces and linear maps.

I.e. defining Hilbert spaces and linear maps ‘from diagrams’ is a conservative extension of string diagrams.

– *completeness* –

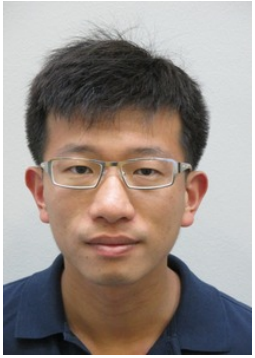
THM. (Selinger, 2008)

An equation between string diagrams holds, if and only if it holds for Hilbert spaces and linear maps.

I.e. defining Hilbert spaces and linear maps ‘from diagrams’ is a conservative extension of string diagrams.

Since last summer, we have:

UNIVERSAL COMPLETENESS!


– *completeness* –

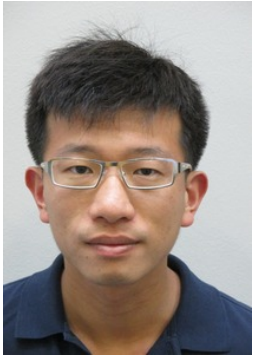
THM. (Selinger, 2008)

An equation between string diagrams holds, if and only if it holds for Hilbert spaces and linear maps.

I.e. defining Hilbert spaces and linear maps ‘from diagrams’ is a conservative extension of string diagrams.

Since last summer, we have:

UNIVERSAL COMPLETENESS!


(see three papers at LiCS this year on ZX-calculus)

— Ch. 4 – Quantum processes —

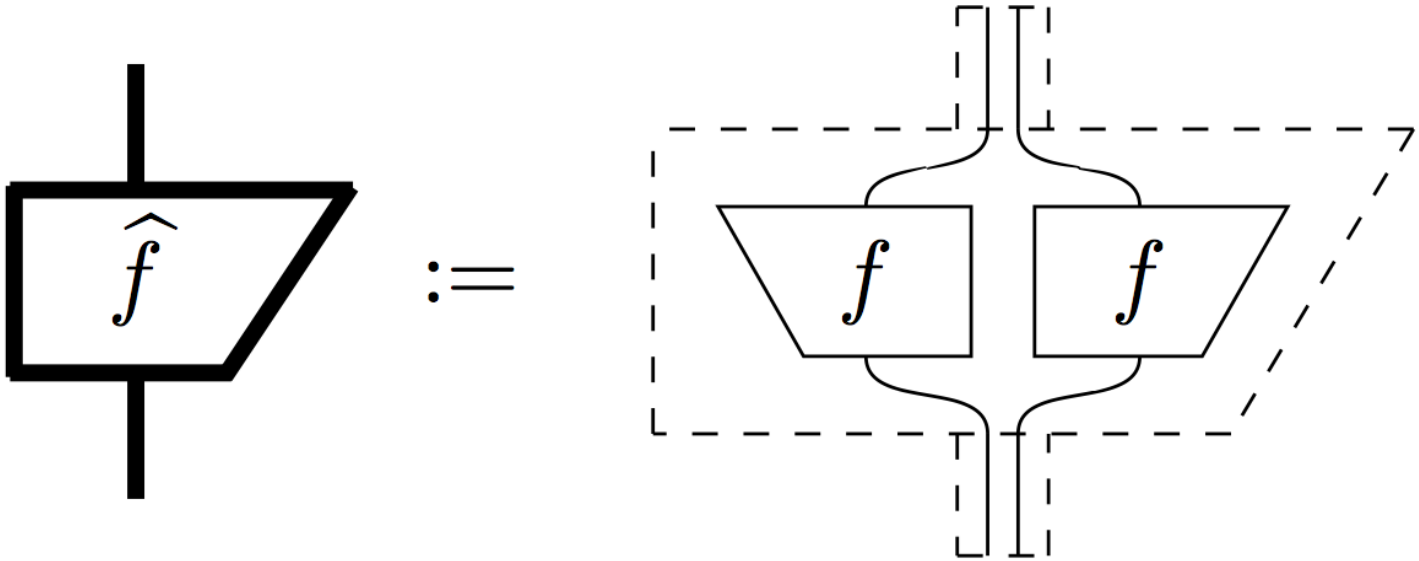
The art of progress is to preserve order amid change, and to preserve change amid order.

— Alfred North Whitehead, Process and Reality, 1929.

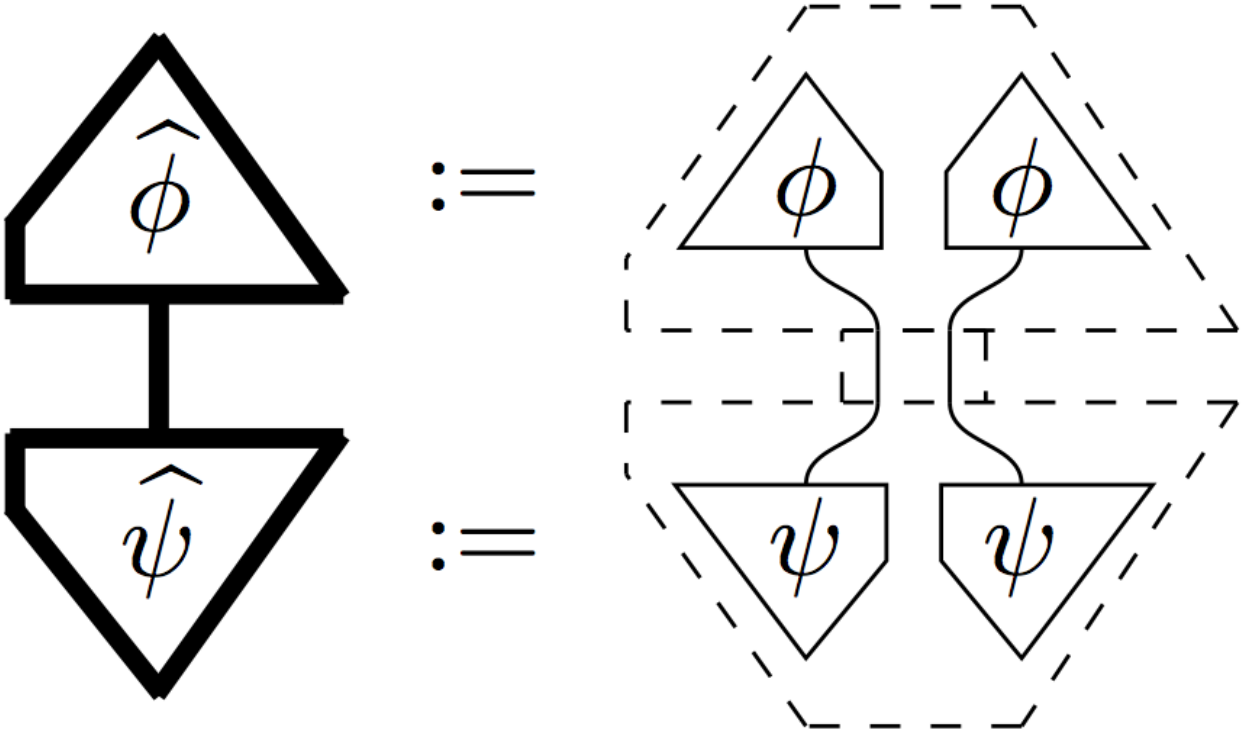
– quantum vs. classical –

$$\frac{\text{classical system}}{\textbf{quantum system}} = \frac{\text{single wire}}{\textbf{double wire}}$$

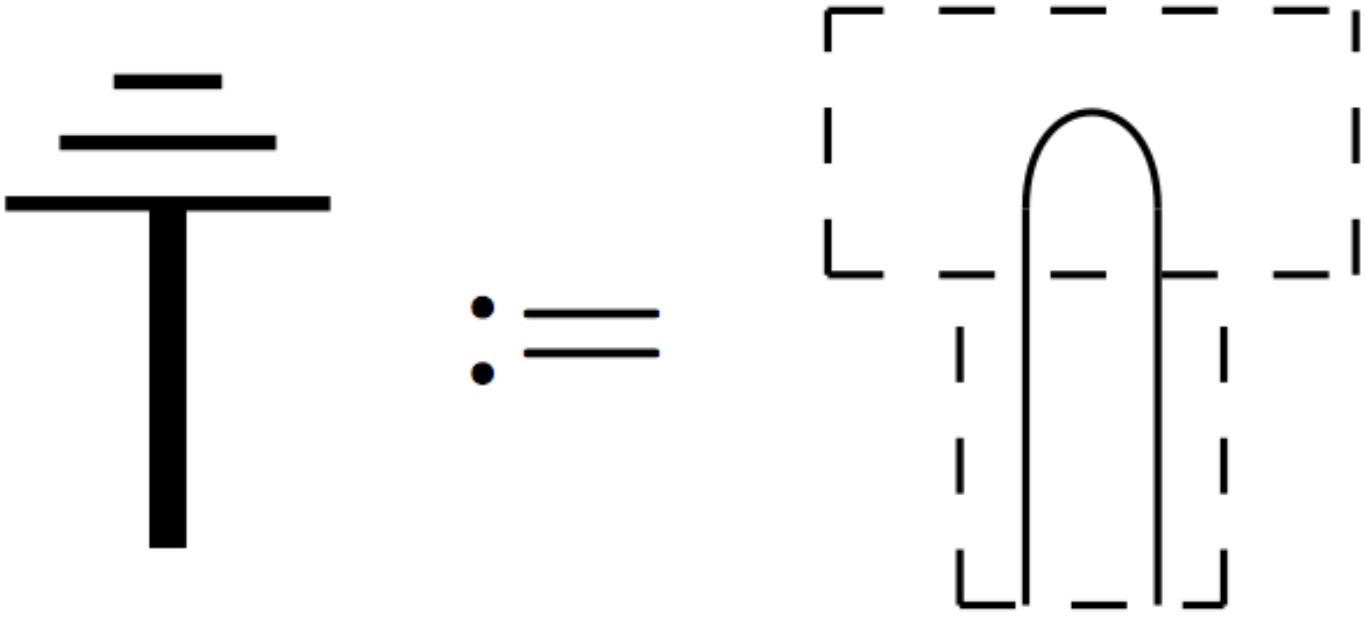
– pure quantum process –



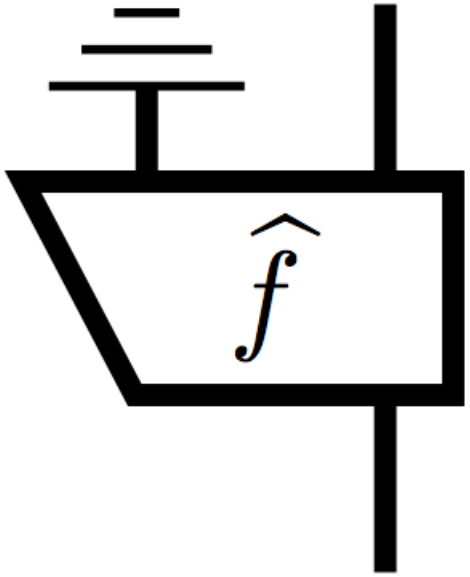
– Born-rule –



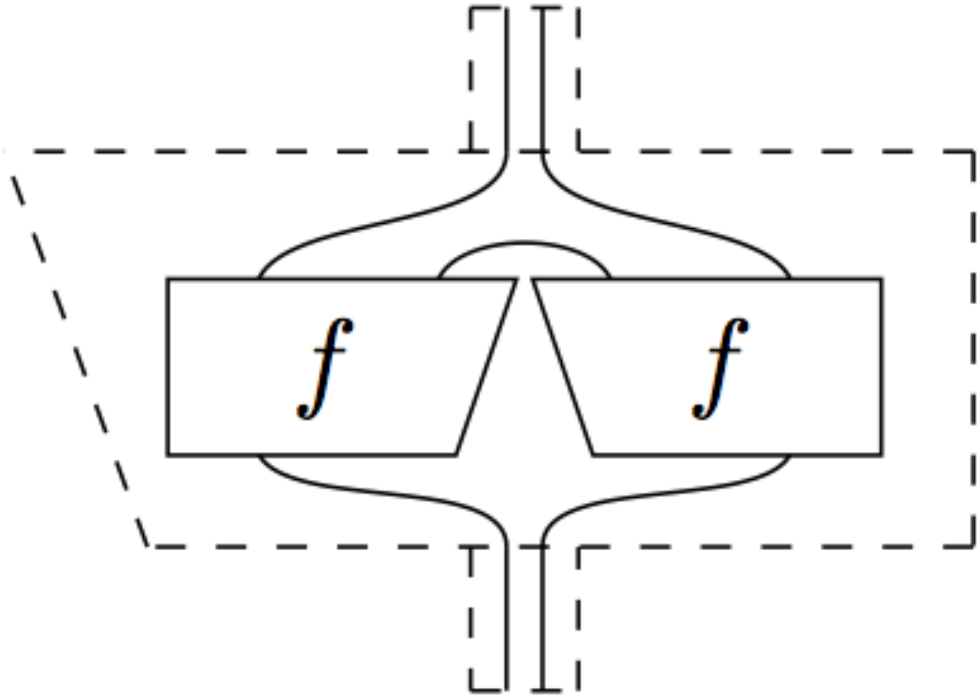
– discarding –



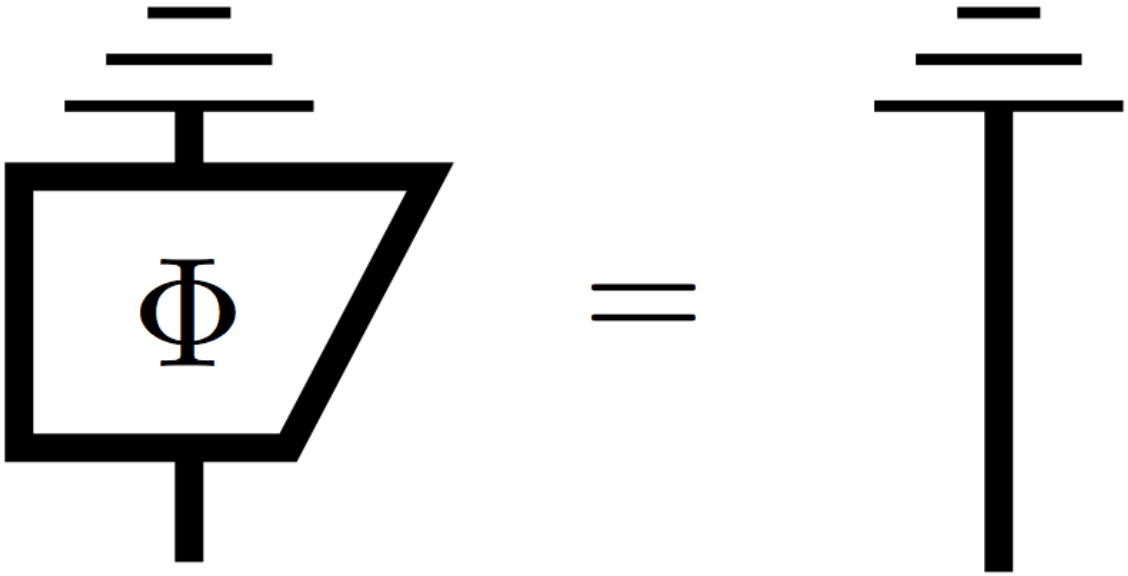
– mixed quantum process –



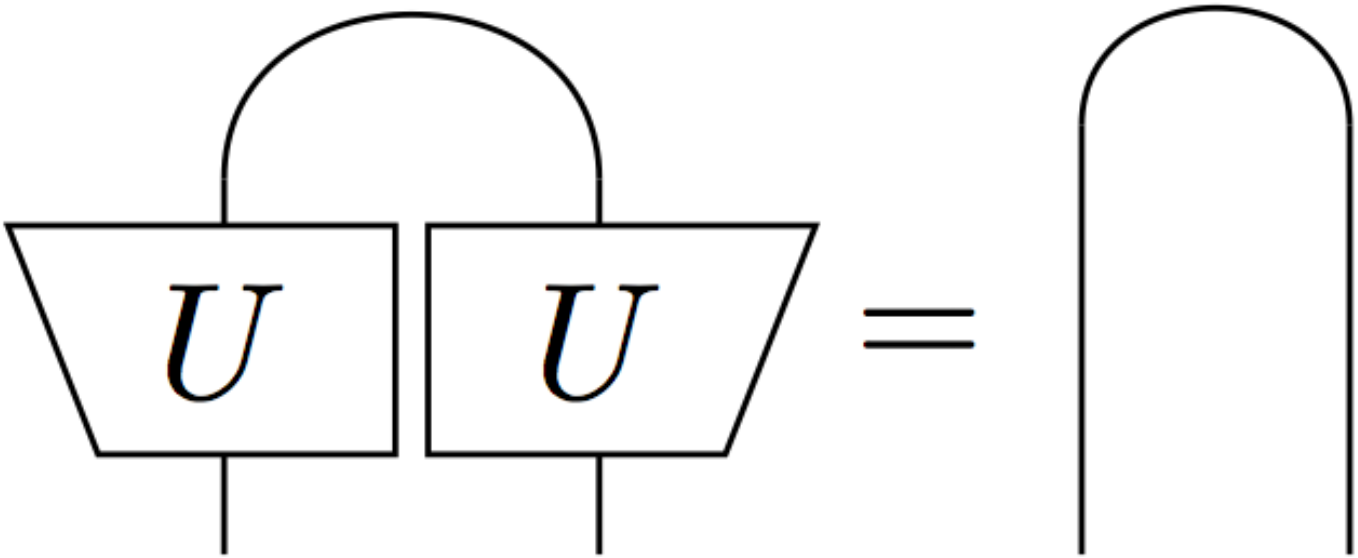
– mixed quantum process –



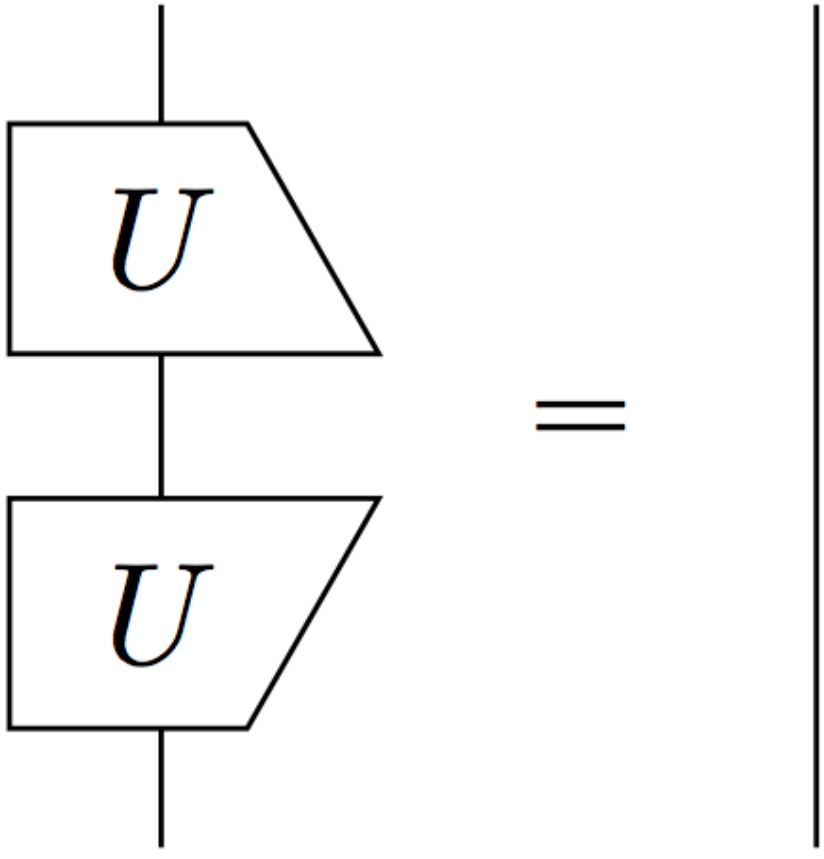
– causality –



– causality –



– causality –

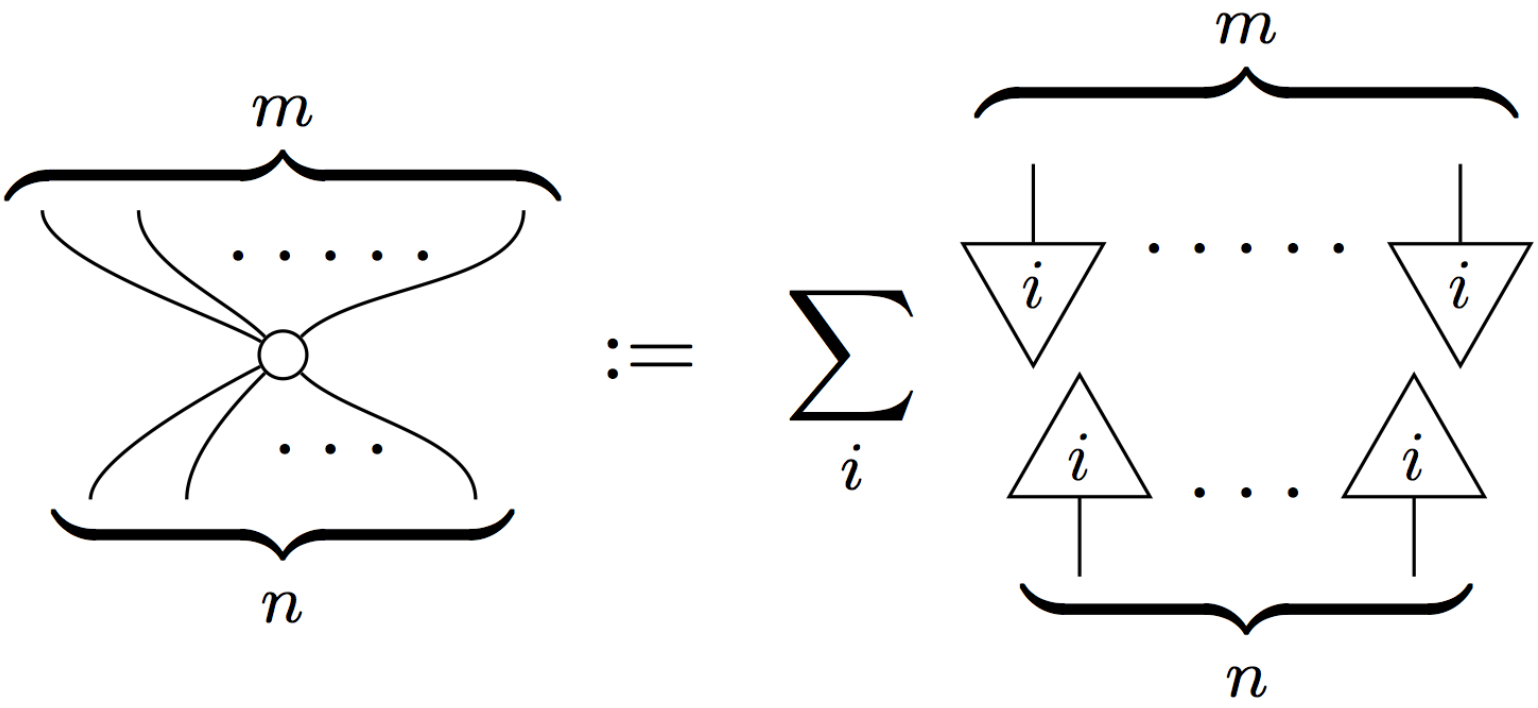


— Ch. 6 – Picturing classical processes —

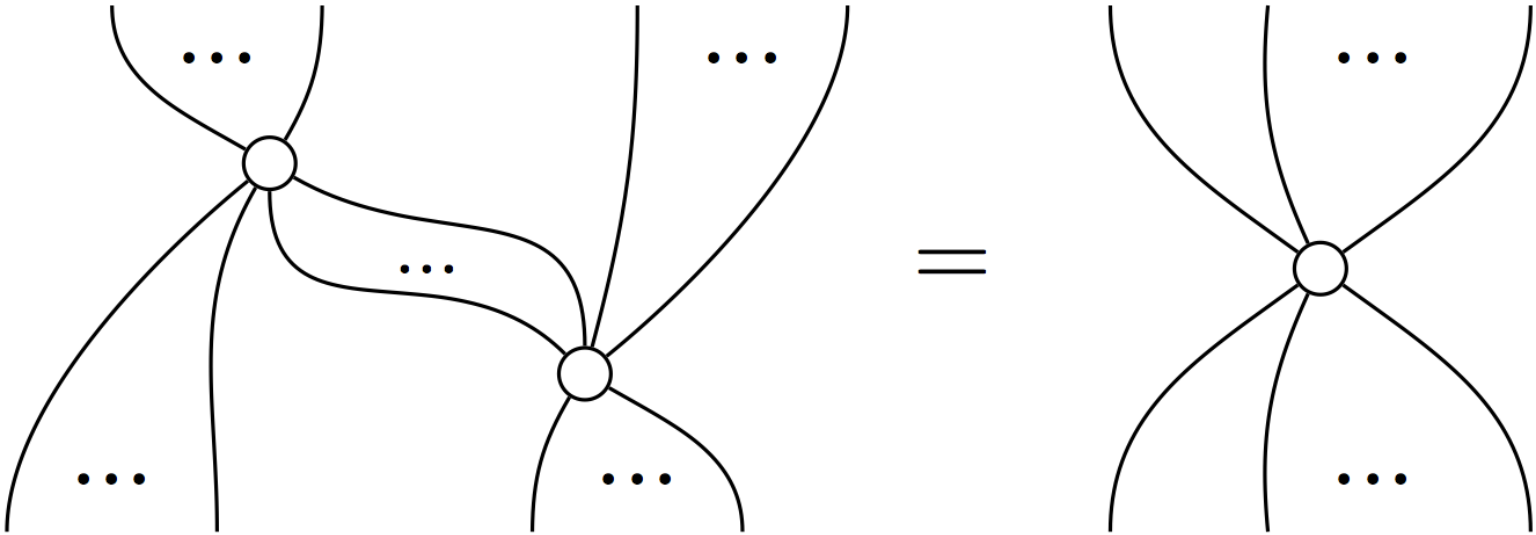
Damn it! I knew she was a monster!
John! Amy! Listen! Guard your buttholes.

— David Wong, This Book Is Full of Spiders, 2012.

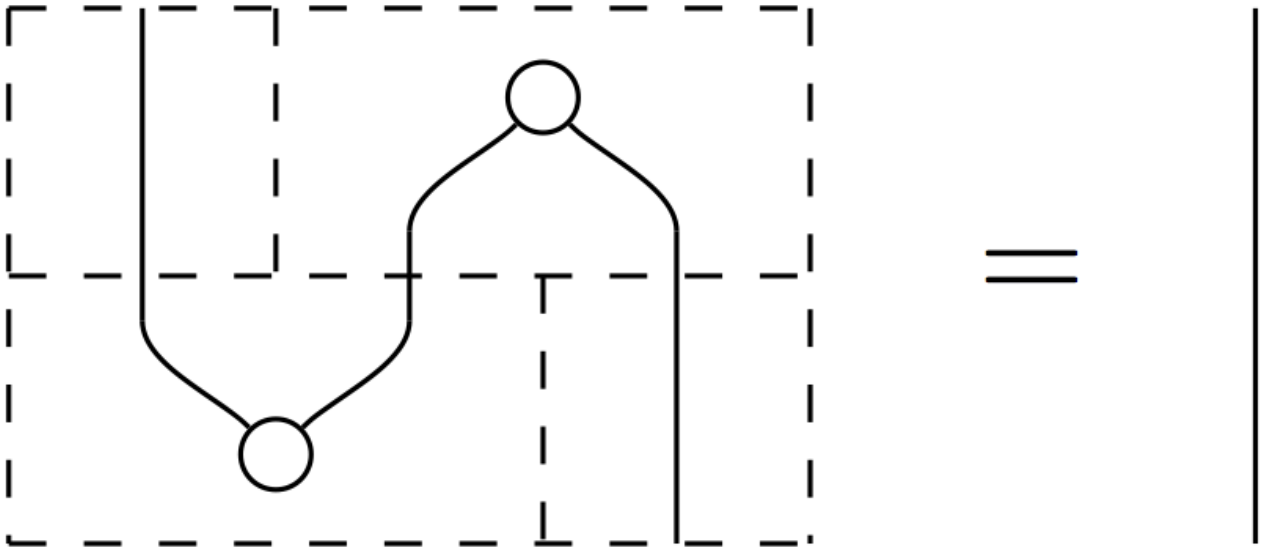
– spiders –



– spiders –

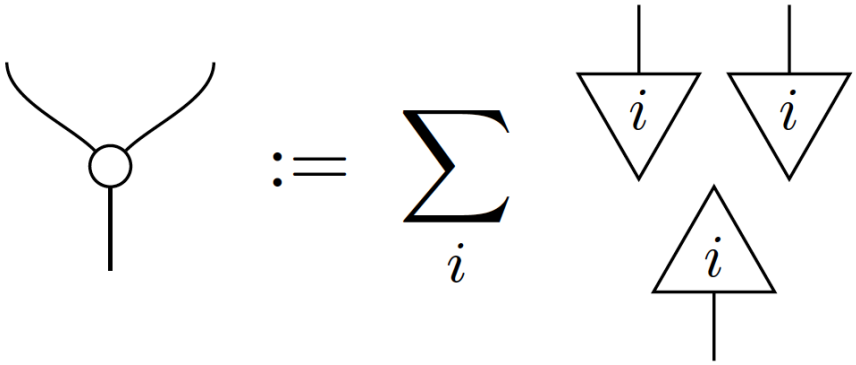


– spiders –

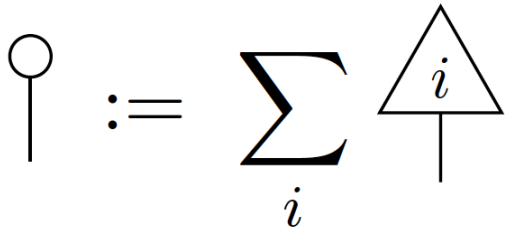


– spiders –

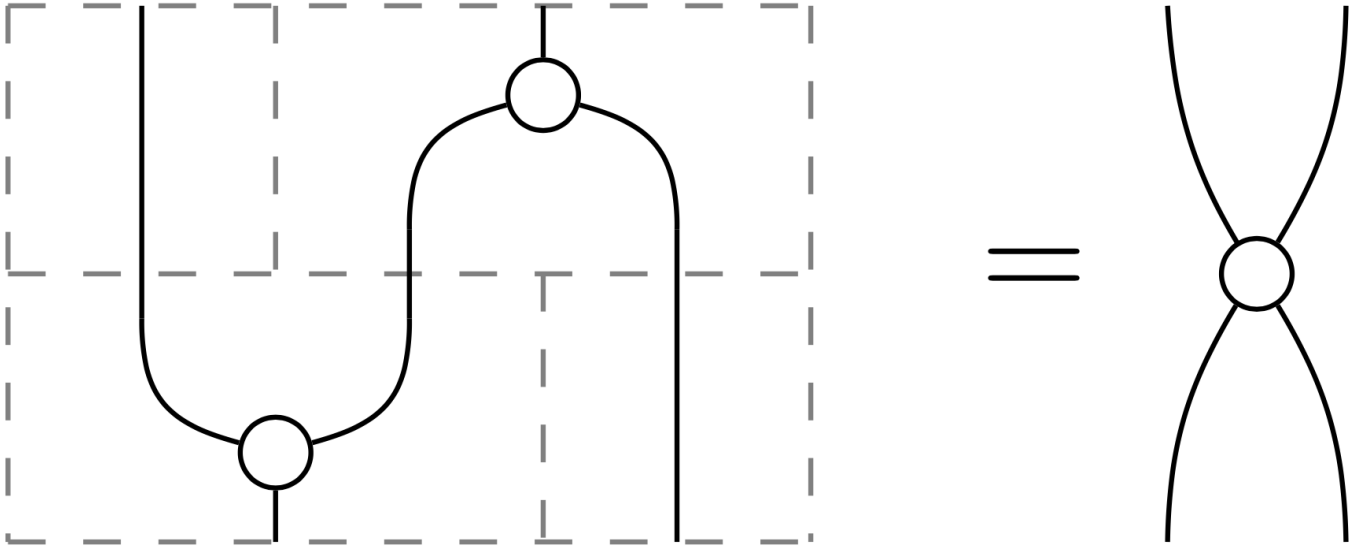
copy :=



delete :=



– spiders –



THM 1. spiders ≡ dag. spec. comm. Frob. algs

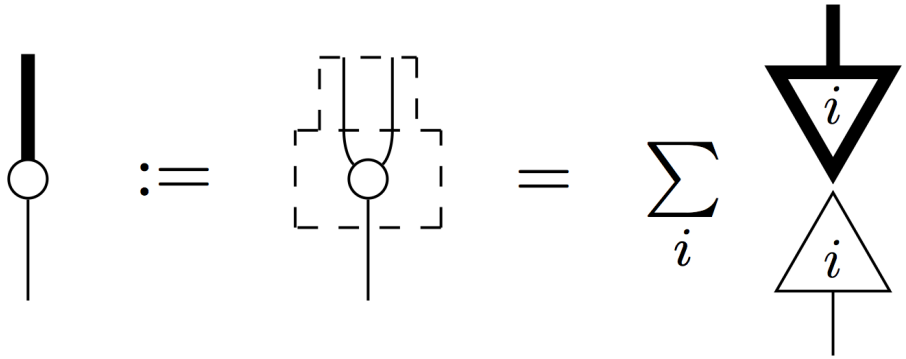
– *spiders* –

THM 1. spiders ≡ dag. spec. comm. Frob. algs

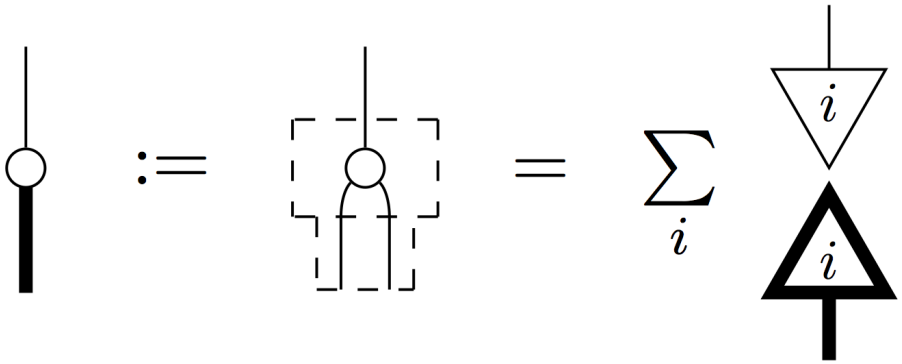
THM 2. FHilb: dag. spec. comm. Frob. algs ≡ ONBs

– spiders –

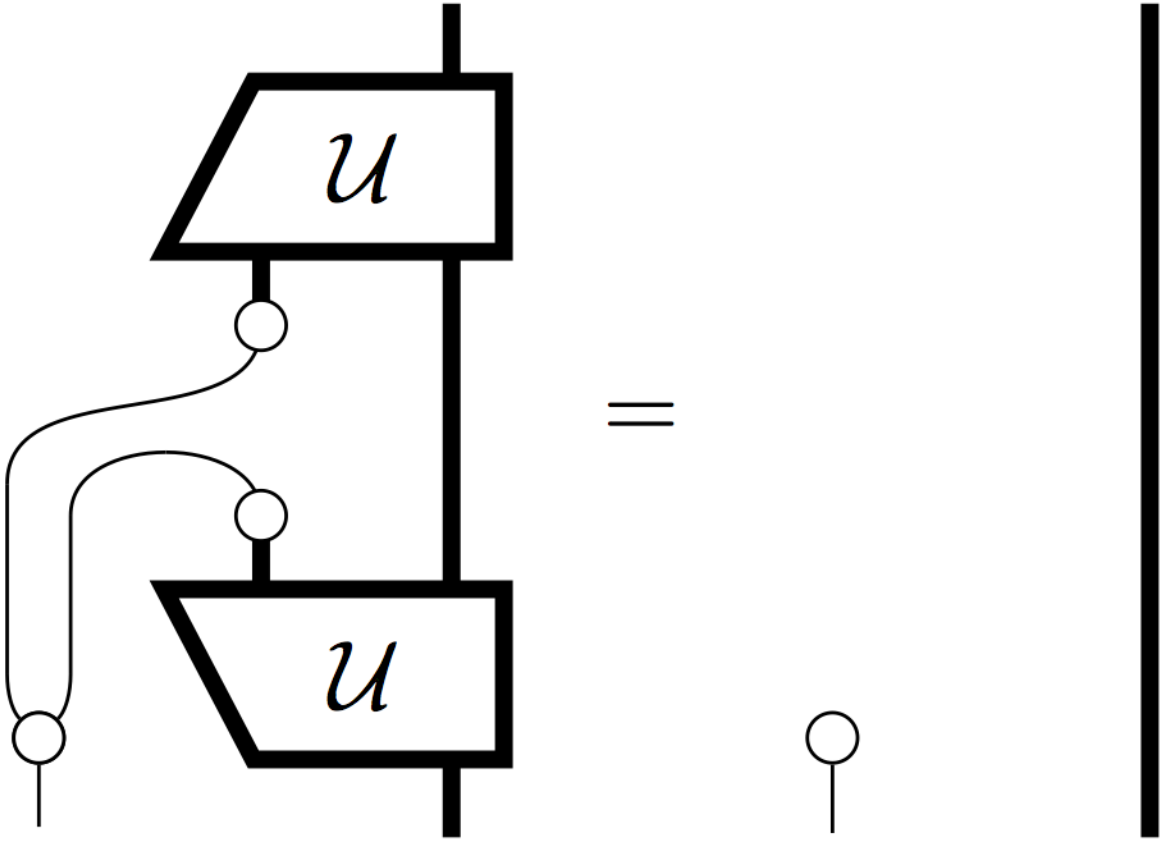
encode :=



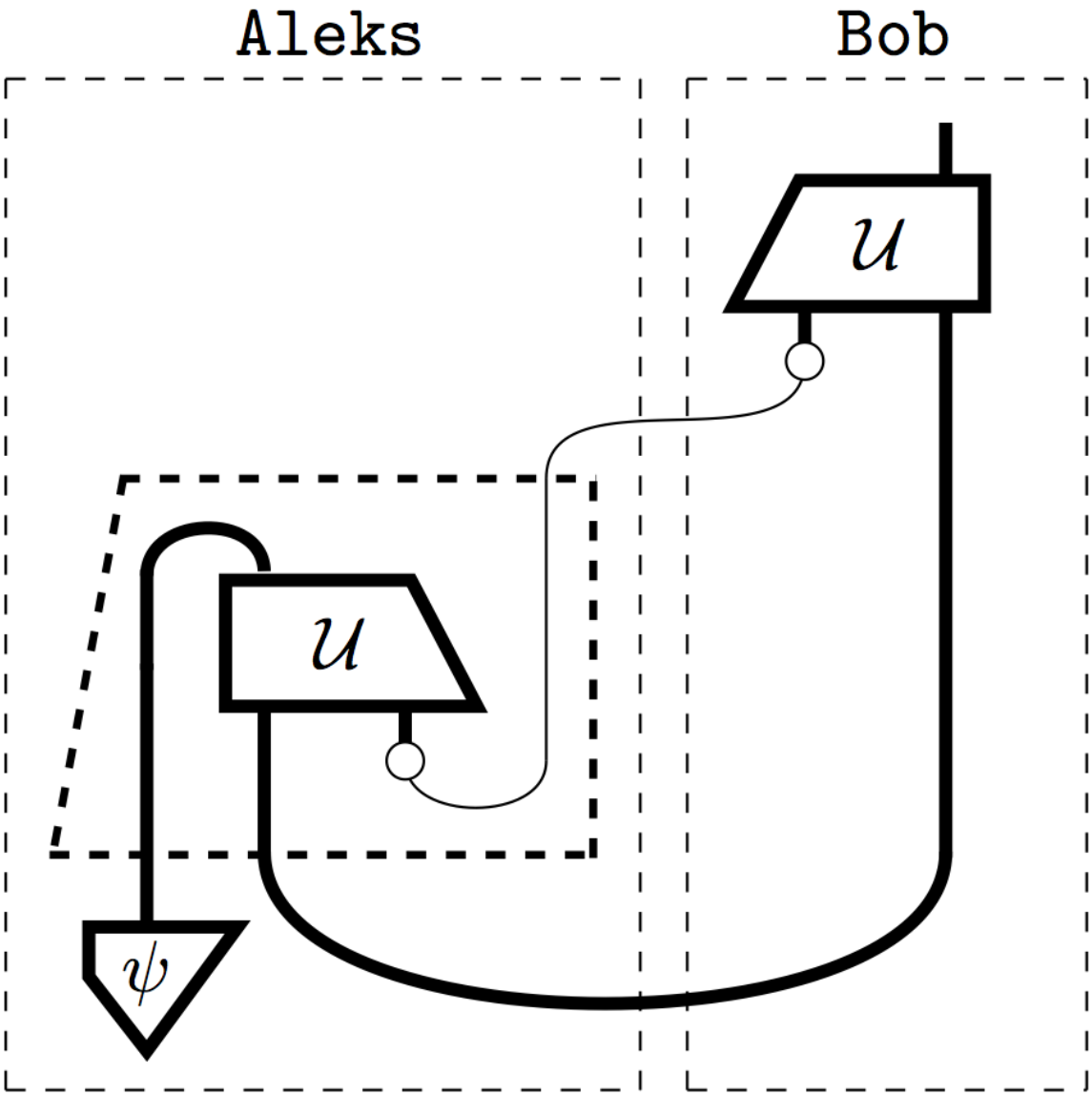
measure :=



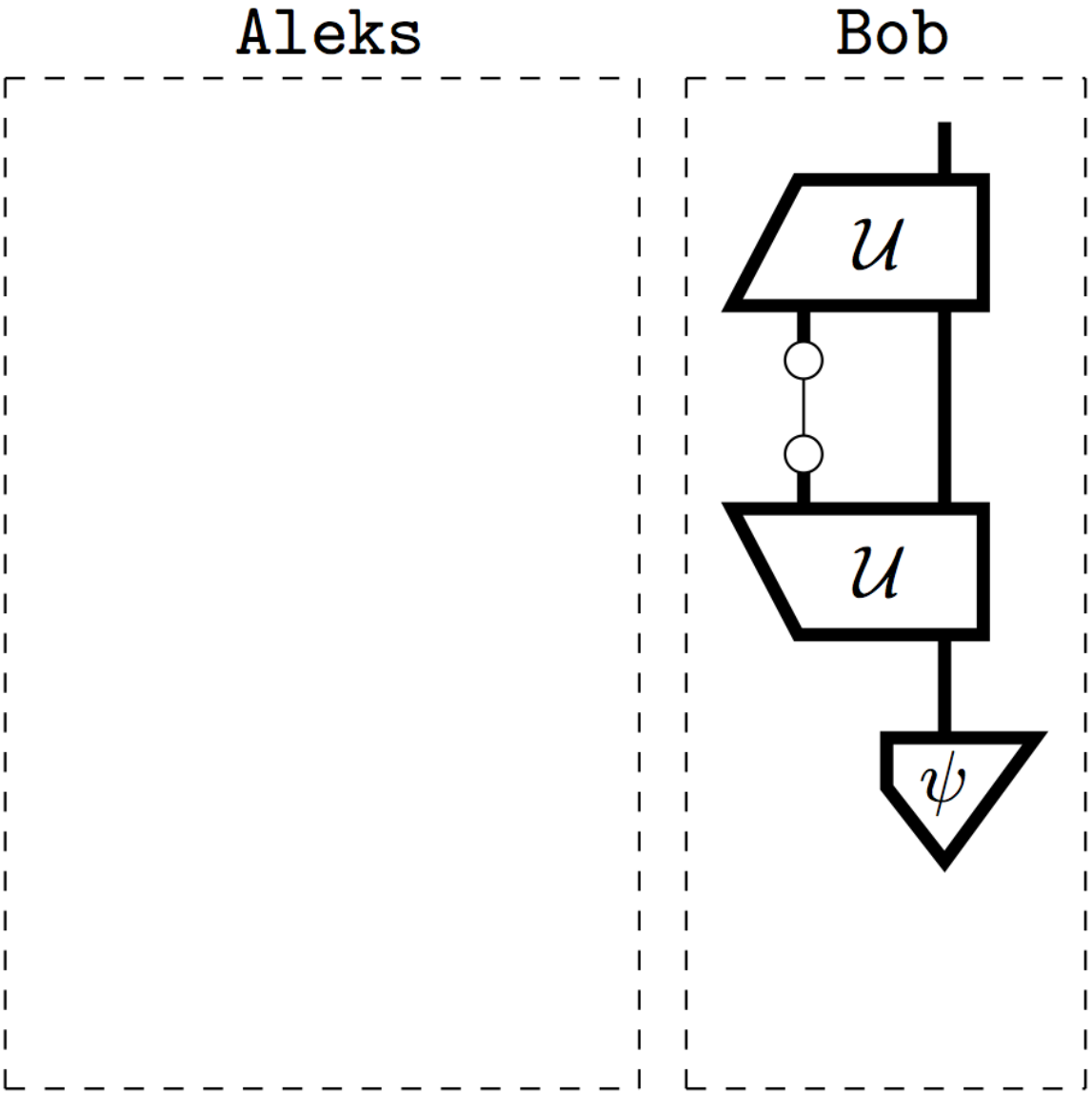
controlled isometry :=



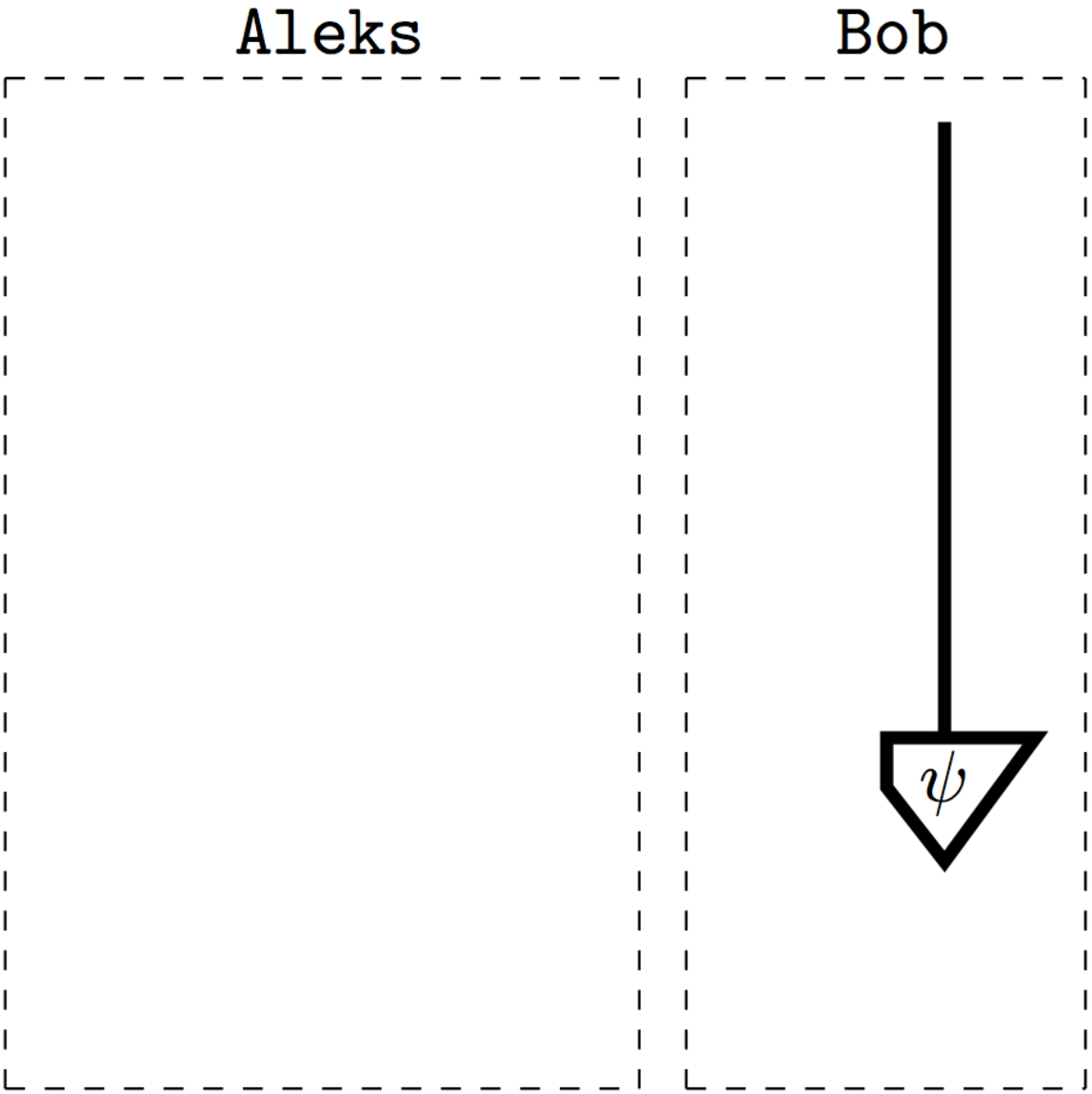
— teleportation with phone call —

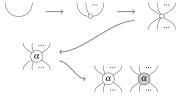


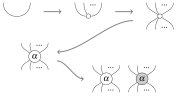
— teleportation with phone call —

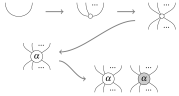


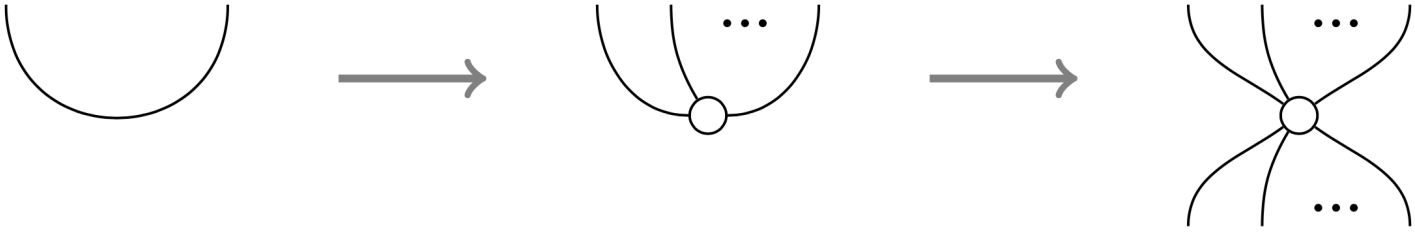
— teleportation with phone call —

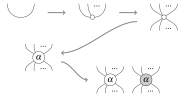


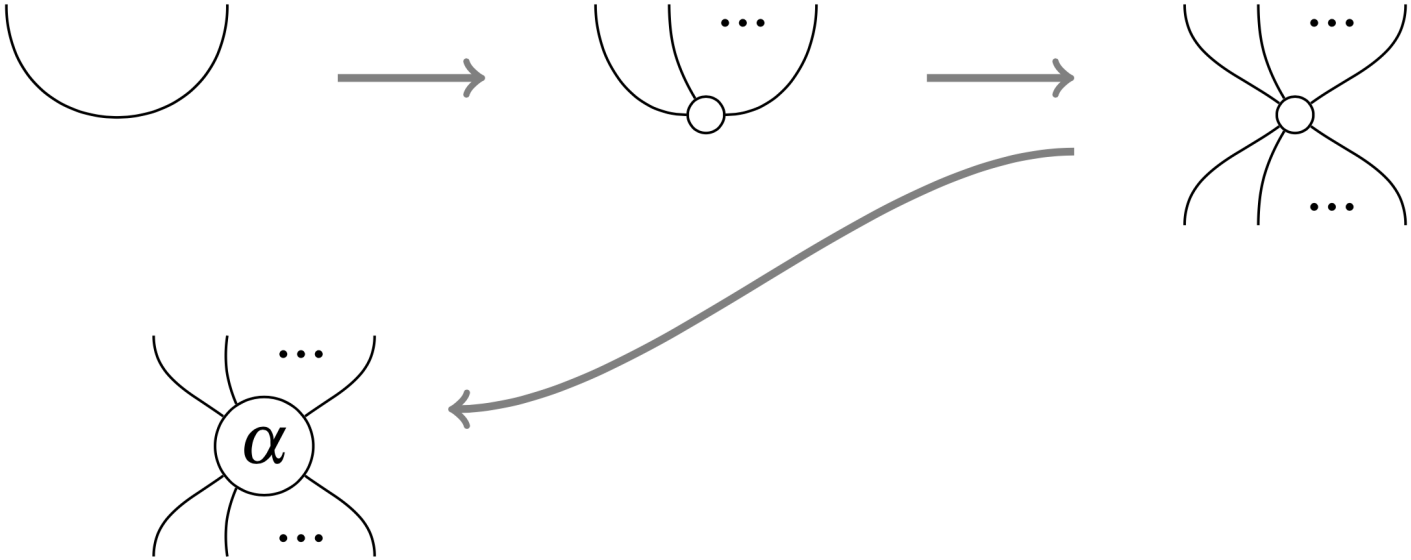
— structural evolution  —

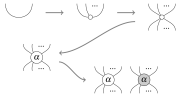
— structural evolution  —

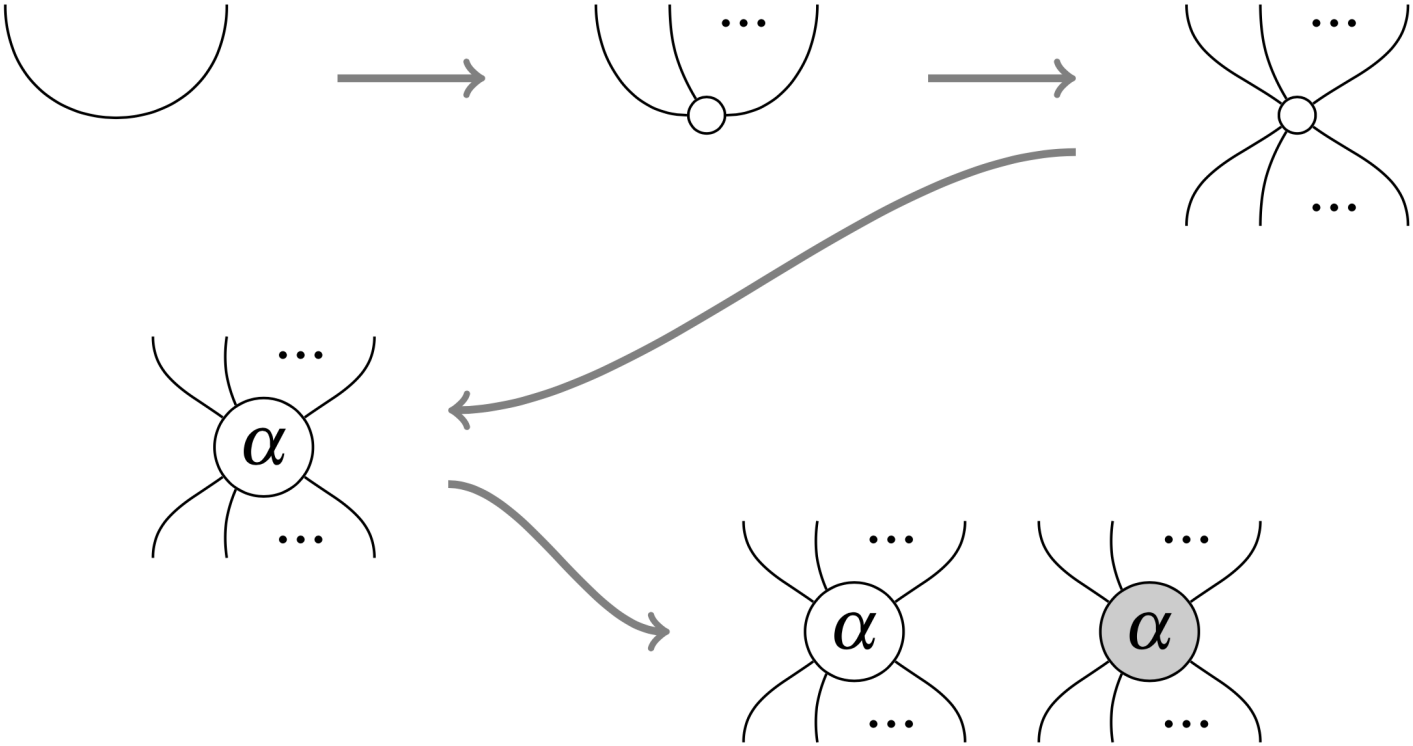
— structural evolution  —



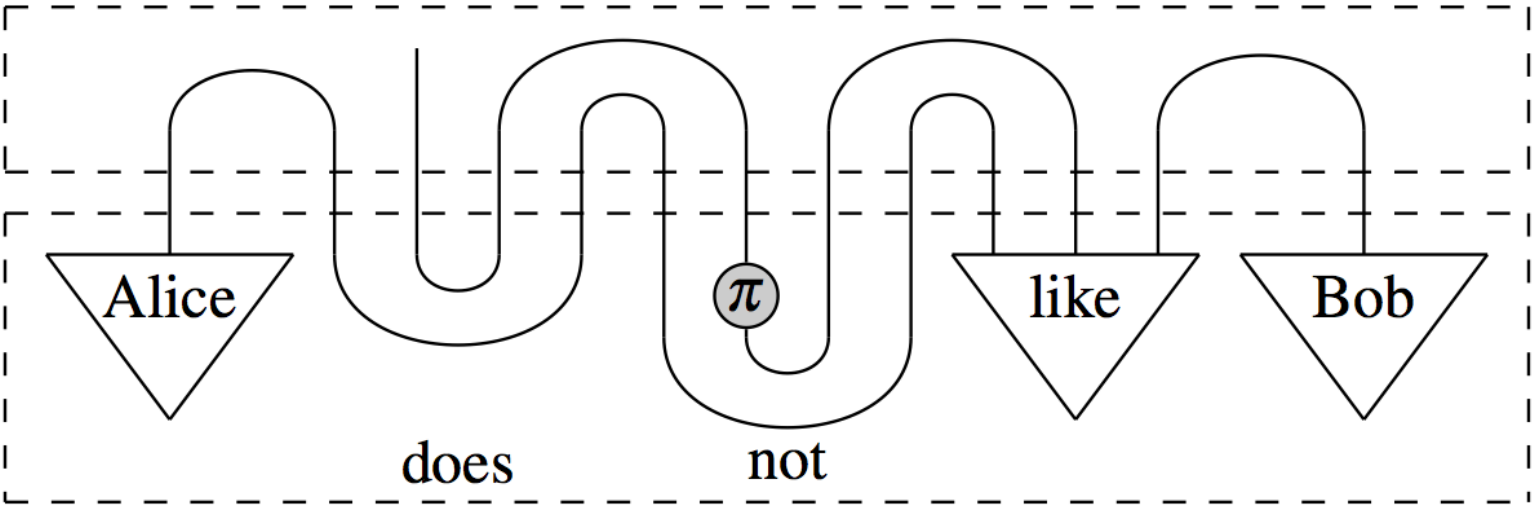
— structural evolution  —

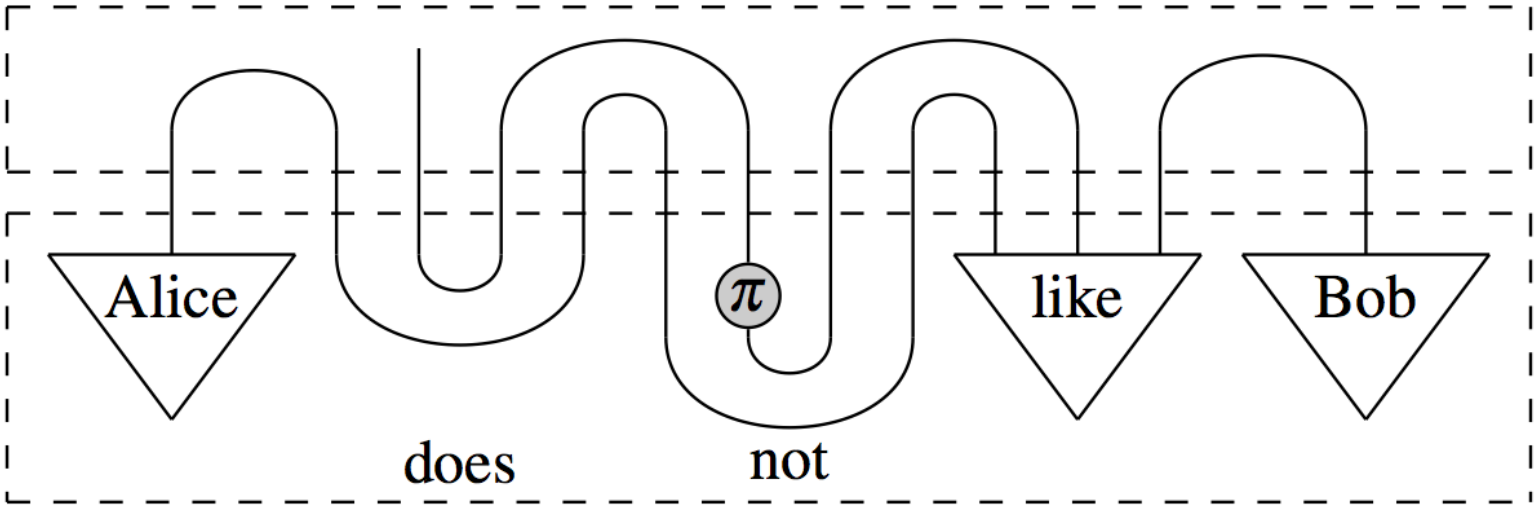


— structural evolution  —



something else

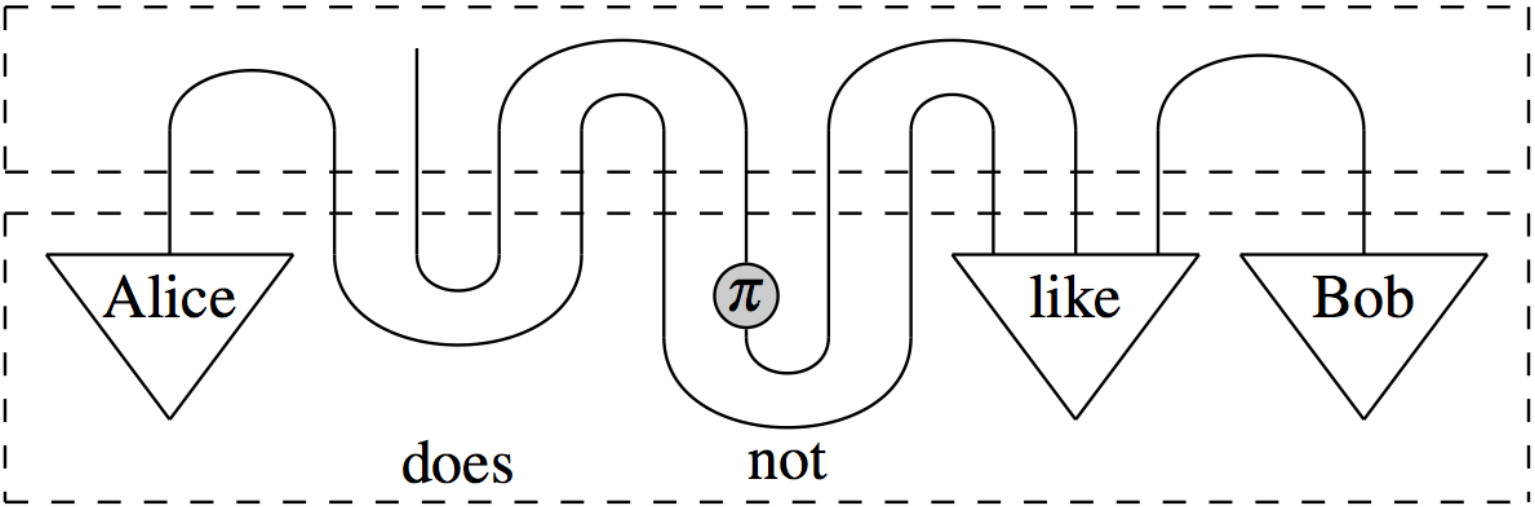

Alice
does
not
π
like
Bob



- Bottom part: meaning
- Top part: grammar

B. Coecke, M. Sadrzadeh & S. Clark (2010) *Mathematical foundations for a compositional distributional model of meaning*. Lambek Festschrift. arXiv:1003.4394

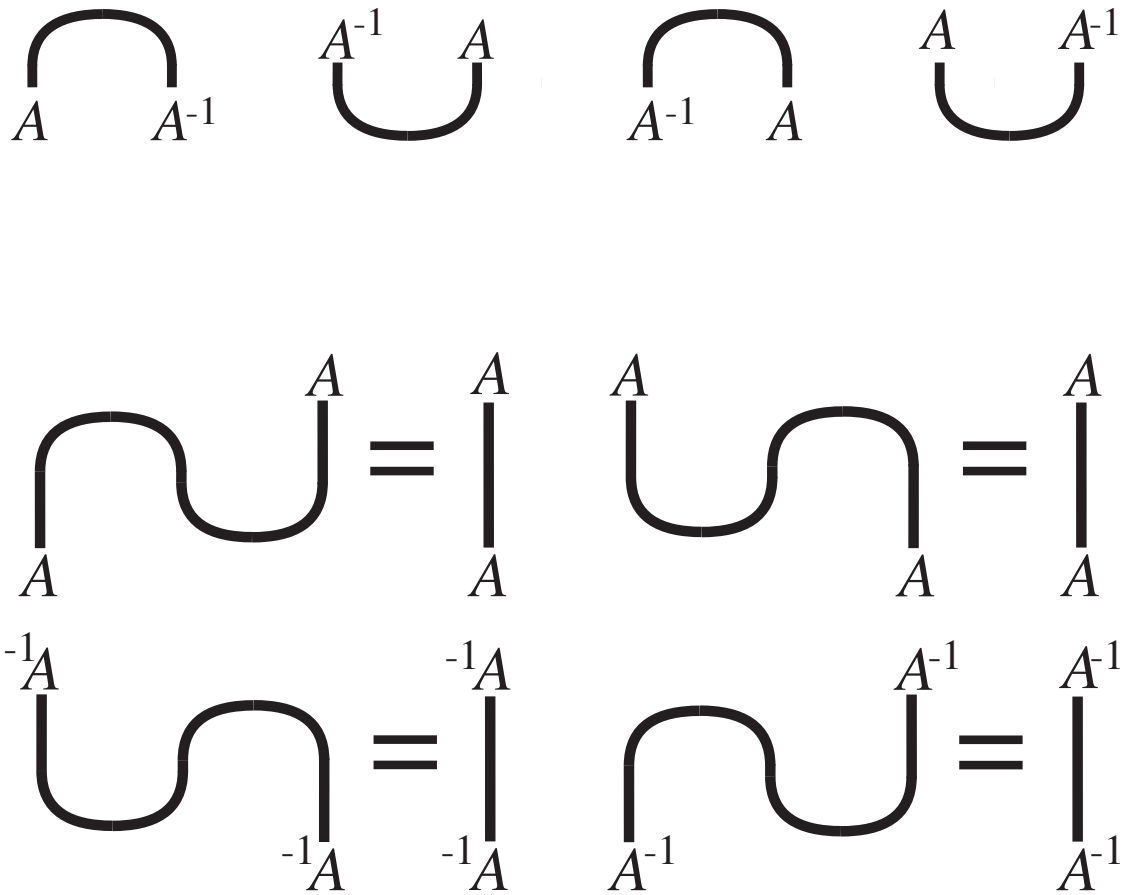
This is why there aren't dictionaries for sentences:



- Bottom part: meaning
- Top part: grammar

B. Coecke, M. Sadrzadeh & S. Clark (2010) *Mathematical foundations for a compositional distributional model of meaning*. Lambek Festschrift. arXiv:1003.4394

Lambek's Pregroups (2000's):



Mathematics of grammar:

For noun type n, verb type is ${}^{-1}n \cdot s \cdot n^{-1}$, so:

Mathematics of grammar:

For noun type n, verb type is ${}^{-1}n \cdot s \cdot n^{-1}$, so:

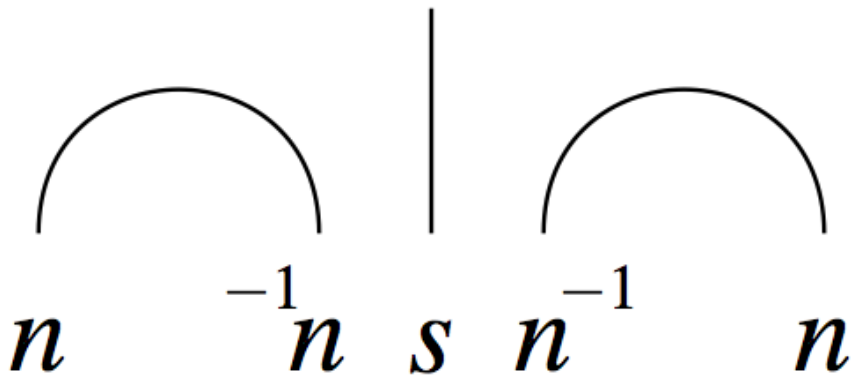
$$n \cdot {}^{-1}n \cdot s \cdot n^{-1} \cdot n \leq 1 \cdot s \cdot 1 \leq s$$

Mathematics of grammar:

For noun type n, verb type is ${}^{-1}n \cdot s \cdot n^{-1}$, so:

$$n \cdot {}^{-1}n \cdot s \cdot n^{-1} \cdot n \leq 1 \cdot s \cdot 1 \leq s$$

As a diagram:

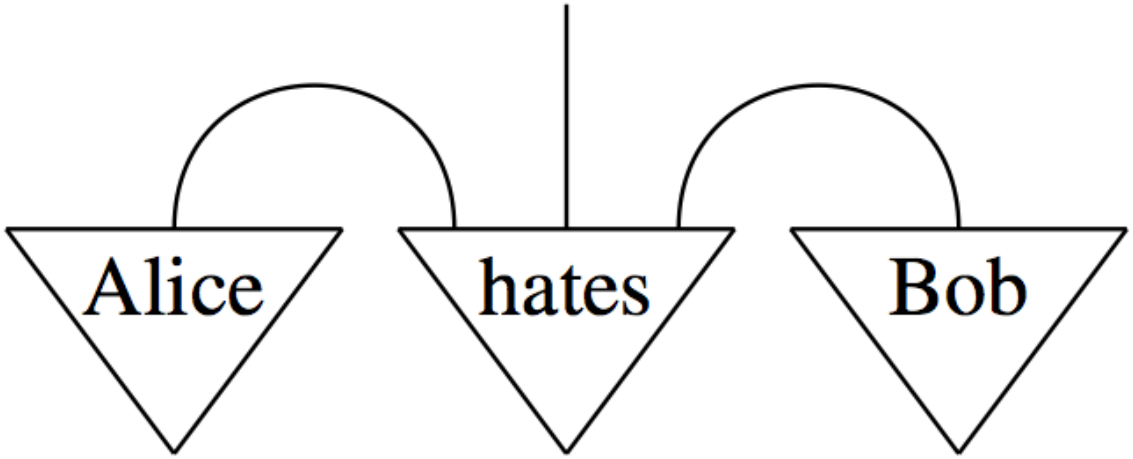


Mathematics of grammar:

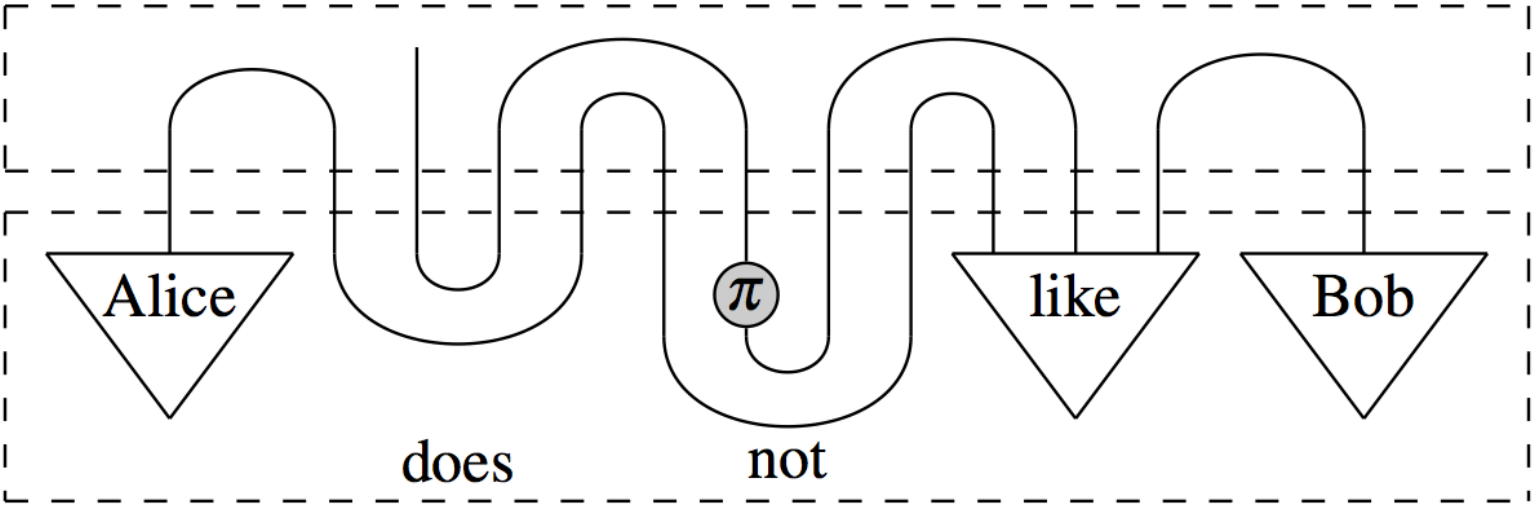
For noun type n, verb type is ${}^{-1}n \cdot s \cdot n^{-1}$, so:

$$n \cdot {}^{-1}n \cdot s \cdot n^{-1} \cdot n \leq 1 \cdot s \cdot 1 \leq s$$

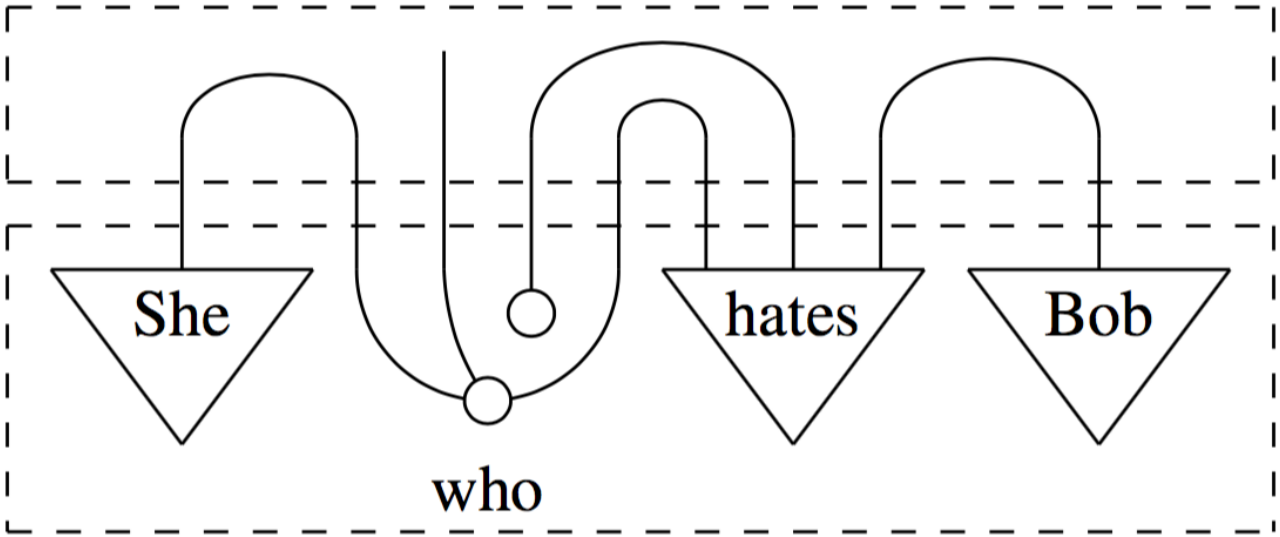
As a diagram:



Logical meanings:



Logical meanings:



M. Sadrzadeh, B. Coecke & S. Clark (2013–2014) *The Frobenius anatomy of word meaning I & II*. Journal of Logic and Computation. arXiv:1404.5278

Interpretation of transitive verb sentence:

- Action which needs two “participants”

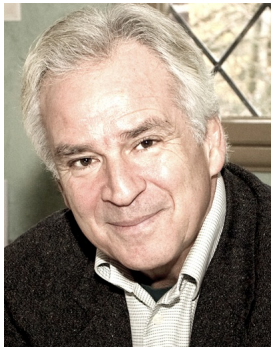
Claim: a lion understands grammar, since is aware of:

- action of hunting
- him/her being hunter
- wants to get a prey

Books by famous developmental psychologist:

- P. Gärdenfors (2004) Conceptual Spaces: The Geometry of Thought. MIT.
- P. Gärdenfors (2014) The Geometry of Meaning. MIT.

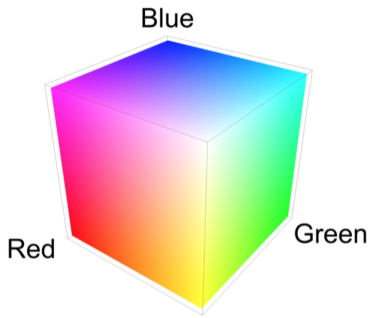
We made it compositional:



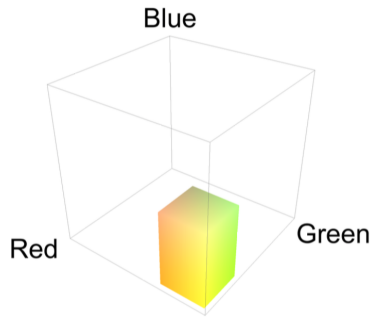
$\hookrightarrow$ **ConvRel**

J. Bolt, B. Coecke, F. Genovese, M. Lewis, D. Marsden & R. Piedeleu (2017) *Interacting Conceptual Spaces I: Grammatical Composition of Concepts*. arXiv:1703.08314

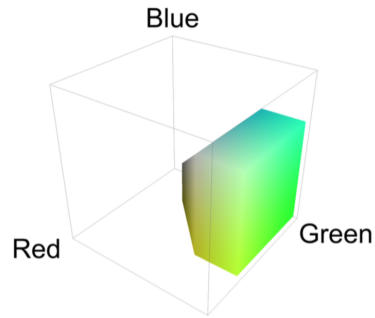
Due to <u>senses mediation</u> and <u>cultural conventions</u>:



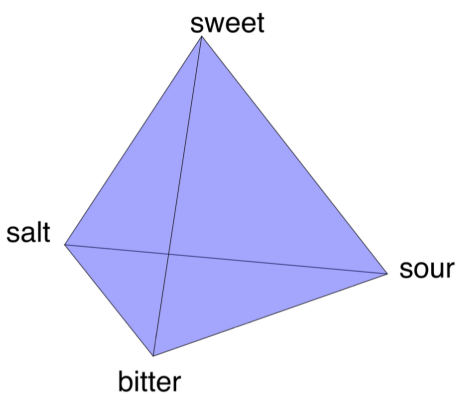
(a) The RGB colour cube



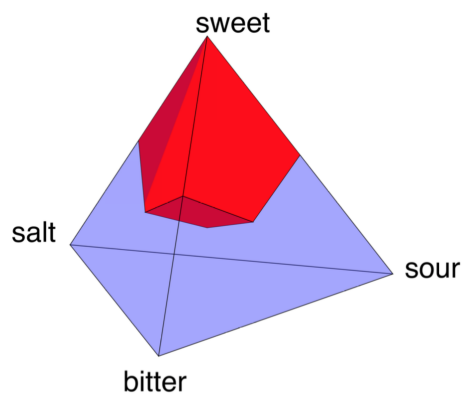
(b) Property p_{yellow}



(c) Property p_{green}



(a) The taste tetrahedron



(b) The property p_{sweet}

Interpretation of wires, boxes, and wirings:

A **convex algebra** is set A and 'mixing' function:

$$\alpha : D(A) \to A$$

with

$$\alpha(|a\rangle) = a \qquad \alpha\left(\sum_{i,j} p_i q_{i,j} |a_{i,j}\rangle\right) = \alpha\left(\sum_i p_i |\alpha(\sum_j q_{i,j} |a_{i,j}\rangle)\rangle\right)$$

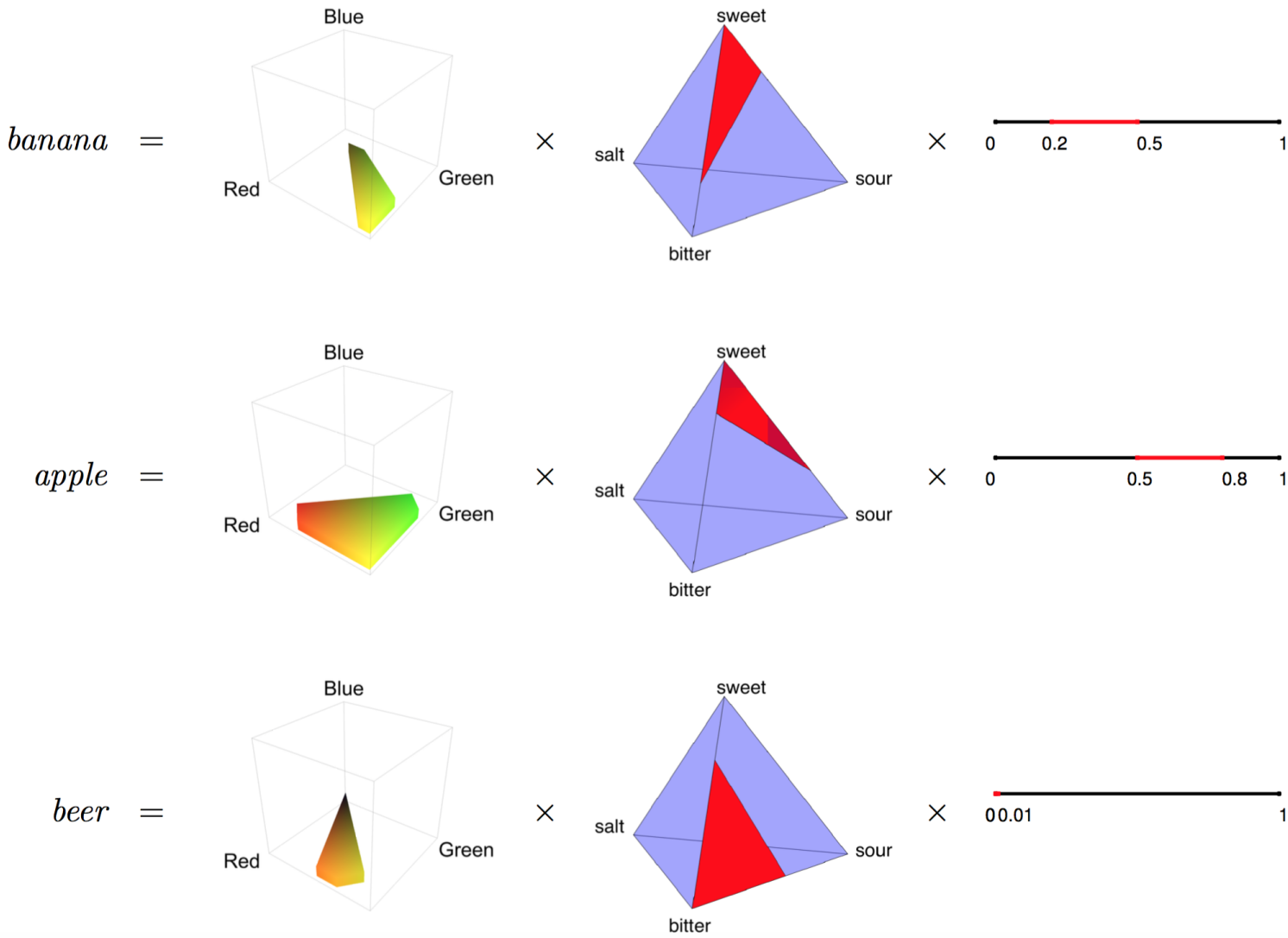
A **convex relation** is relation that 'commutes with mixtures':

$$(\forall i. R(a_i, b_i)) \Rightarrow R\left(\sum_i p_i a_i, \sum_i p_i b_i\right)$$

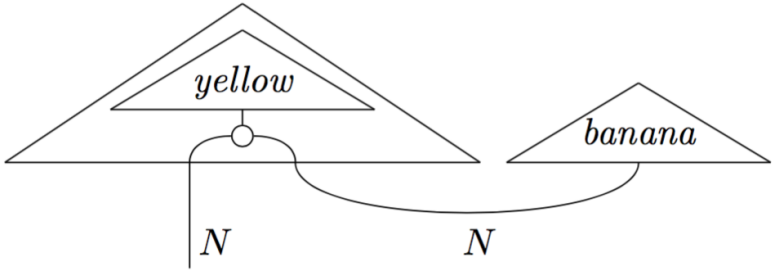
RELEVANT STRUCTURES:

- tensor := cartesian
- cups := also like in Rel
- spiders := ONB ones in Rel

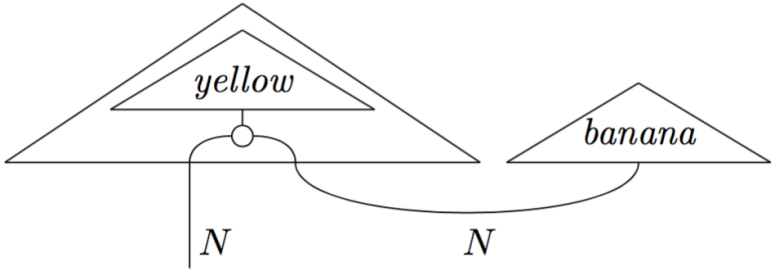
$$N_{food} = N_{colour} \otimes N_{taste} \otimes N_{texture}$$



Phrase example:

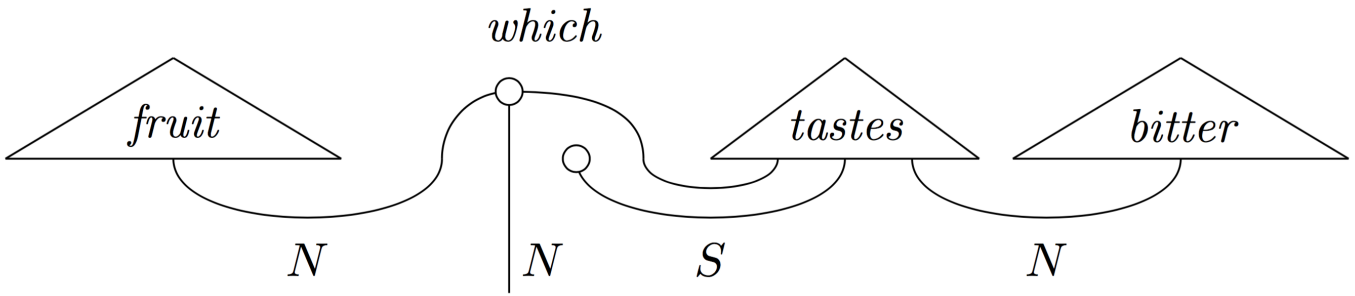


Phrase example:

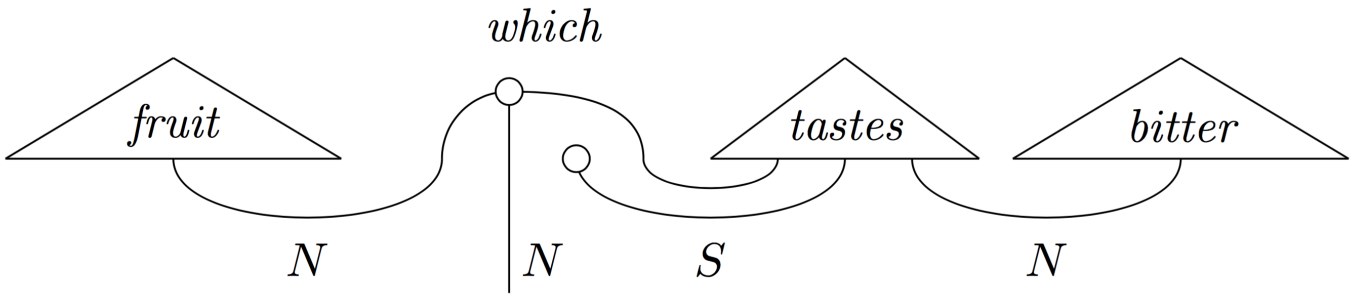


$\Rightarrow$ intersection of yellow & banana

Relative pronoun example:

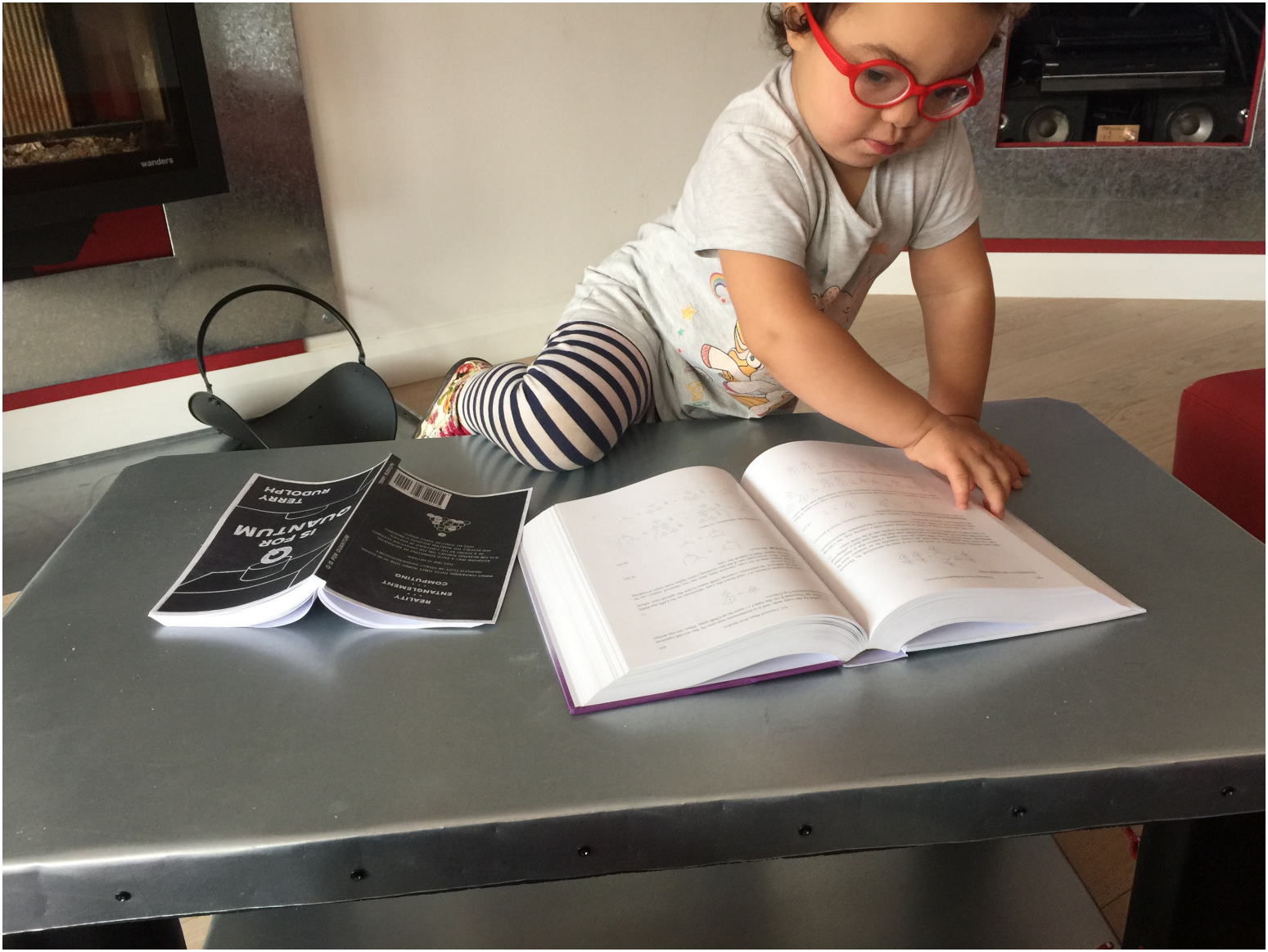


Relative pronoun example:

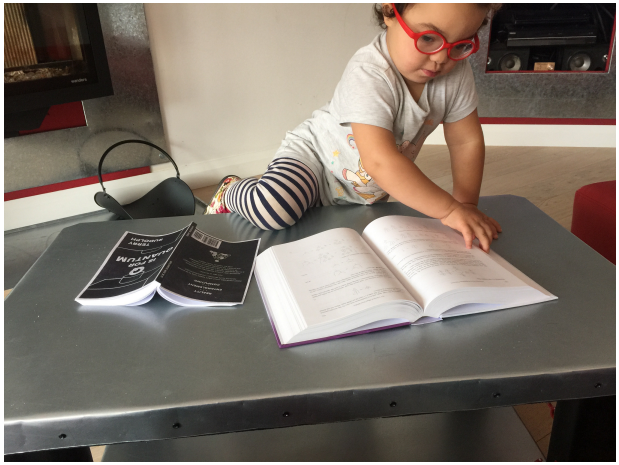


$\Rightarrow$ intersection of fruit & tastes bitter

Any age restrictions?



EXPERIMENTS THIS SUMMER !



KIDS OUTPERFORM OXFORD STUDENTS AND DISCOVER QUANTUM FEATURES THAT TOOK TOP SCIENTISTS 60y