(In)finite Trace Equivalence of Probabilistic Transition Systems

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- ▷ Section 1 : Definition of the trace semantics of probabilistic transition systems

  From the work of Kerstan (2013).

- ▷ Section 2 : Coalgebraic construction of the trace semantics

  Original work with inspiration from Jacobs/Silva/Sokolova (2015).

- ▷ Section 3 : Algorithm for trace equivalence

  Original work with inspiration from Bonchi/Pous (2013).

- ▷ Section 4 : Continuous trace semantics

Section 1

Definition of the trace semantics

Definition (Distribution functor)

The distribution functor $\mathcal{D} : \mathbf{Set} \to \mathbf{Set}$ is defined by

$$\mathcal{D}(X) = \{p : X \to [0,1] \mid \sum_{x \in X} p(x) = 1\}$$

$$\mathcal{D}(f)(u) = y \mapsto \sum_{x \in f^{-1}(\{y\})} u(x)$$

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Definition (Subdistribution functor)

$$\mathcal{S}(X) = \{p : X \to [0,1] \mid \sum_{x \in X} p(x) \leq 1\}$$

Finite alphabet A
Notation : $1 = \{*\}$ (termination singleton).

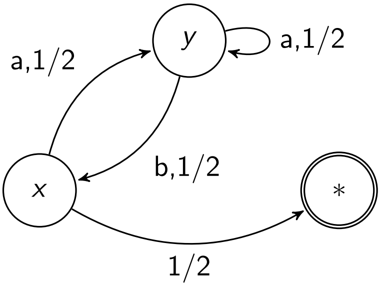
Definition

A probabilistic transition system (PTS) α is a coalgebra for the functor $\mathcal{D}(A \times - + 1)$, i.e.,

$$\alpha : X \to \mathcal{D}(A \times X + 1)$$

- $\alpha(x)$ is a probability on $A \times X + 1$.
- Termination probability is $\alpha(x)(*)$.
- Transition $x \xrightarrow{a} y$ probability is $\alpha(x)(a, y)$.

$A = \{a, b\}$, $X = \{x, y\}$ $\alpha : X \to \mathcal{D}(A \times X + 1)$



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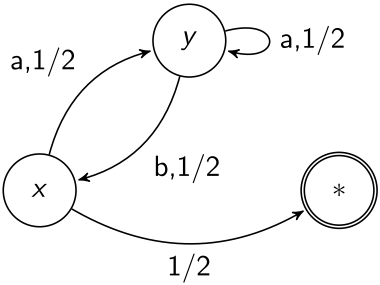

$$[\![x]\!](\varepsilon) = \frac{1}{2}$$

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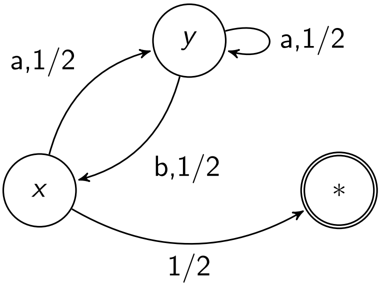

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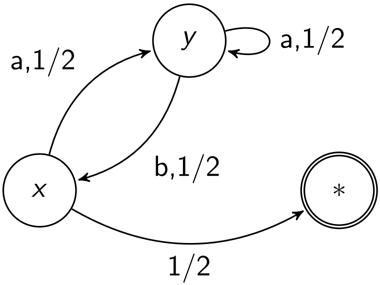

$[\![x]\!](\varepsilon) = \frac{1}{2}$ $\qquad$ $[\![y]\!](b) = \frac{1}{2} \cdot [\![x]\!](\varepsilon) = \frac{1}{4}$ $\qquad$ $[\![y]\!](abab) = \frac{1}{32}$

Definition (Trace semantics of α)

By induction on words $w \in A^*$.

$$[\![x]\!](\varepsilon) = \alpha(x)(*) \qquad [\![x]\!](aw) = \sum_{y \in X} \alpha(x)(a, y) \cdot [\![y]\!](w)$$

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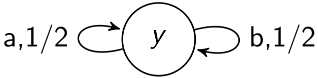

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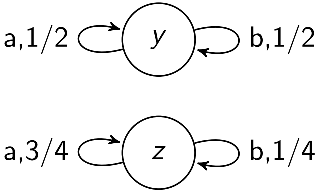

$A^\infty = A^* \cup A^\omega$ with concatenation $A^* \times A^\infty \to A^\infty$

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Definition (Measurable sets of words)

Let $S_\infty = \{\emptyset\} \cup \{\{w\} \mid w \in A^*\} \cup \{wA^\infty \mid w \in A^*\}$. The σ-algebra of ***measurable sets of words*** *is defined by $\Sigma_{A^\infty} = \sigma_{A^\infty}(S_\infty)$.*

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- $\{w\}$ for any $w \in A^\infty$
- Any countable language
- Any language of finite words
- $\emptyset, A^*, A^\omega, A^\infty$
- Concatenation LS $(L \subseteq A^*,\ M \in \Sigma_{A^\infty})$

Theorem (Extension)

Let $m : S_\infty \to \mathbb{R}_+$ be a map such that $m(\emptyset) = 0$. Are equivalent :

(i) *There exists a unique measure $\tilde{m}$ on Σ_{A^∞} s.t. $\tilde{m}_{|S_\infty} = m$.*

(ii) *For all $w \in A^*$, $m(wA^\infty) = m(w) + \sum_{a \in A} m(waA^\infty)$*

Definition (Trace semantics of α)

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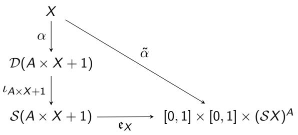
$$[\![y]\!](bA^\infty) = \frac{1}{2} \neq \frac{1}{4} = [\![z]\!](bA^\infty)$$

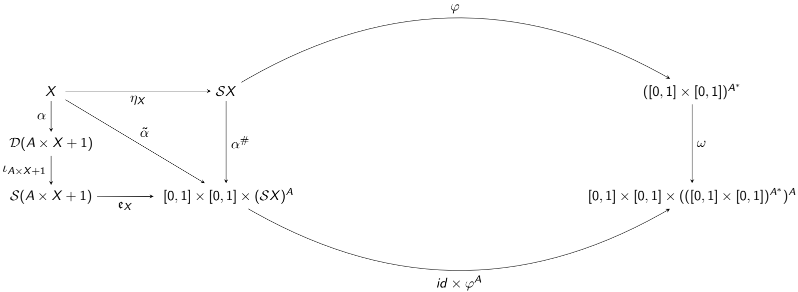
Section 2

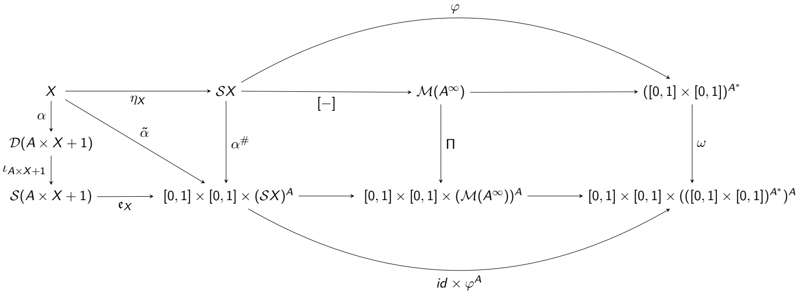
Coalgebraic construction of the trace semantics

Our construction

$$\begin{array}{c} X \\ \alpha \downarrow \\ \mathcal{D}(A \times X + 1) \end{array}$$


X
α
$\tilde{\alpha}$
$\mathcal{D}(A \times X + 1)$
$\iota_{A \times X+1}$
$\mathcal{S}(A \times X + 1)$
$\mathfrak{e}_X$
$[0,1] \times [0,1] \times (\mathcal{S}X)^A$


φ
X
η_X
$\mathcal{S}X$
$([0,1] \times [0,1])^{A^*}$
α
$\tilde{\alpha}$
$\alpha^{\#}$
ω
$\mathcal{D}(A \times X + 1)$
$\iota_{A \times X + 1}$
$\mathcal{S}(A \times X + 1)$
$\mathfrak{e}_X$
$[0,1] \times [0,1] \times (\mathcal{S}X)^A$
$[0,1] \times [0,1] \times (([0,1] \times [0,1])^{A^*})^A$
$id \times \varphi^A$


φ
X
η_X
$\mathcal{S}X$
$[-]$
$\mathcal{M}(A^\infty)$
$([0,1] \times [0,1])^{A^*}$
α
$\tilde{\alpha}$
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Π
ω
$\mathcal{D}(A \times X + 1)$
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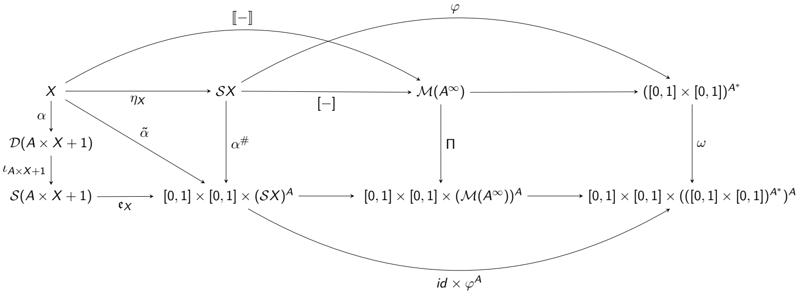
⟦−⟧
φ
X
SX
M(A^∞)
([0,1] × [0,1])^{A*}
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α#
Π
ω
D(A × X + 1)
ι_{A×X+1}
S(A × X + 1)
𝔢_X
[0,1] × [0,1] × (SX)^A
[0,1] × [0,1] × (M(A^∞))^A
[0,1] × [0,1] × (([0,1] × [0,1])^{A*})^A
id × φ^A

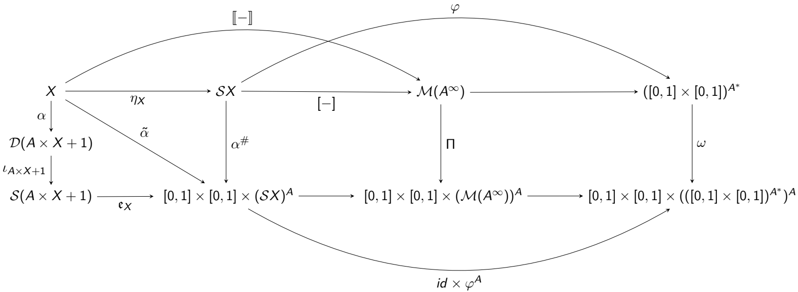
$\mathcal{M}(A^\infty)$ is the space of sub-probability measures on $(A^\infty, \Sigma_{A^\infty})$.

Definition

The measure coalgebra is defined for all $m \in \mathcal{M}(A^\infty)$ *by*

$$\Pi(m) = \langle m(A^\infty), m(\varepsilon), a \mapsto m_a \rangle$$

- $m(A^\infty)$ is the total mass
- $m(\varepsilon)$ is the termination mass
- The measure derivative $m_a \in \mathcal{M}(A^\infty)$ is defined by $m_a(S) = m(aS)$ for all measurable S.


$[\![-]\!]$
φ
X
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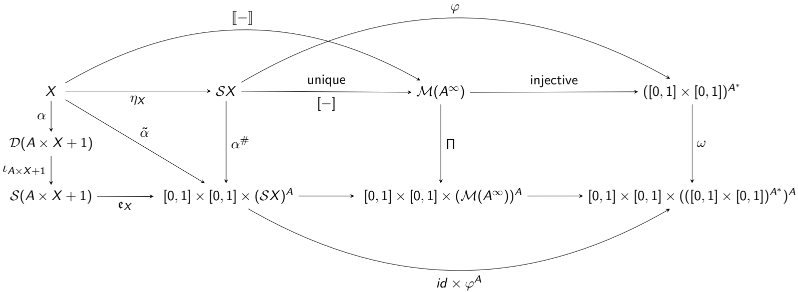
$[\![-]\!]$
φ
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$\mathcal{S}X$
unique
$\mathcal{M}(A^\infty)$
injective
$([0,1]\times[0,1])^{A^*}$
η_X
$[-]$
α
$\tilde{\alpha}$
$\alpha^\#$
Π
ω
$\mathcal{D}(A\times X+1)$
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$[0,1]\times[0,1]\times(([0,1]\times[0,1])^{A^*})^A$
$id\times\varphi^A$

Theorem (Coincidence with the former trace semantics)

Let $\alpha : X \to \mathcal{D}(A \times X + 1)$. The morphism $[\![-]\!]$ obtained via this construction satisfies

$$[\![x]\!](\varepsilon) = \alpha(x)(*) \qquad [\![x]\!](aw) = \sum_{y \in X} \alpha(x)(a, y) \cdot [\![y]\!](w)$$

$$[\![x]\!](\varepsilon A^\infty) = 1 \qquad [\![x]\!](awA^\infty) = \sum_{y \in X} \alpha(x)(a, y) \cdot [\![y]\!](wA^\infty)$$

Section 3

Algorithm for trace equivalence

$$[\![x]\!] = [\![y]\!]\,?$$

Take a PTS

$$\alpha : X \to \mathcal{D}(A \times X + 1)$$

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Determinize it into the Moore automaton

$$\overline{\alpha} = \langle \overline{\alpha}_{\oplus}, \overline{\alpha}_{*}, a \mapsto \overline{\alpha}_a \rangle : \mathbb{R}^X_\omega \to \mathbb{R} \times \mathbb{R} \times \left(\mathbb{R}^X_\omega\right)^A$$

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- Output $\overline{\alpha}_{\oplus} : \mathbb{R}^X \to [0,1]$ (total mass)
- Output $\overline{\alpha}_{*} : \mathbb{R}^X \to [0,1]$ (termination mass)
- (Deterministic) Transitions $\overline{\alpha}_a : \mathbb{R}^X \to \mathbb{R}^X$

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$$\overline{\alpha}_\oplus(u) = \sum_{x \in X} u(x)$$

$$\overline{\alpha}_*(u) = \sum_{x \in X} u(x) \cdot \alpha(x)(*)$$

$$\overline{\alpha}_a(u) = y \mapsto \sum_{x \in X} u(x) \cdot \alpha(x)(a, y)$$

Definition (Congruence closure)

The congruence closure of $R \subseteq \mathbb{R}_\omega^X \times \mathbb{R}_\omega^X$ *is the least relation such that*

- $R \subseteq c(R)$
- $c(R)$ *is an equivalence relation*
- $c(R)$ *is closed under linear combinations*

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- $c(R)$ *is closed under linear combinations*

A bisimulation up-to congruence is a relation $R \subseteq \mathbb{R}_\omega^X \times \mathbb{R}_\omega^X$ such that for all $(u, v) \in R$,

$$\overline{\alpha}_\oplus(u) = \overline{\alpha}_\oplus(v)$$

$$\overline{\alpha}_*(u) = \overline{\alpha}_*(v)$$

$$(\overline{\alpha}_a(u), \overline{\alpha}_a(v)) \in c(R)$$

Theorem

For any $x, y \in X$, $[\![x]\!] = [\![y]\!]$ iff there exists a bisimulation up-to congruence $R \subseteq \mathbb{R}_\omega^X \times \mathbb{R}_\omega^X$ such that $(\delta_x, \delta_y) \in R$.

Theorem

For any $x, y \in X$, $[\![x]\!] = [\![y]\!]$ iff there exists a bisimulation up-to congruence $R \subseteq \mathbb{R}_\omega^X \times \mathbb{R}_\omega^X$ such that $(\delta_x, \delta_y) \in R$.

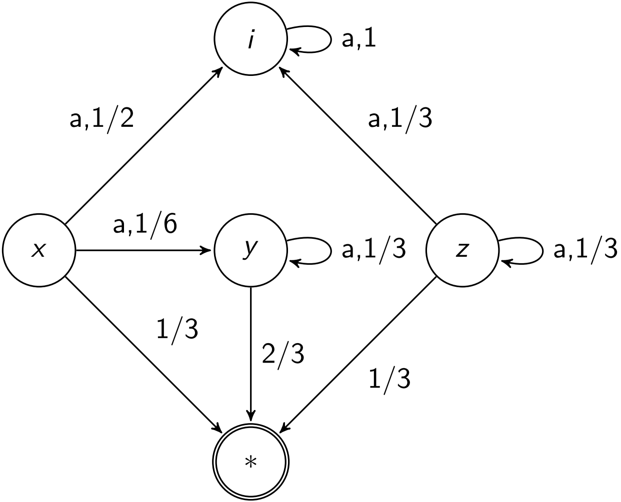
$\delta_x : y \mapsto \delta_{x,y}$

$$\mathtt{HKC}^{\infty}(x, y)$$

(1) $R := \emptyset$; $todo := \emptyset$
(2) insert (δ_x, δ_y) into $todo$
(3) **while** $todo$ is not empty **do**
(3.1) extract (u, v) from $todo$
(3.2) **if** $(u, v) \in c(R)$ **then** **continue**
(3.3) **if** $\overline{\alpha}_{\oplus}(u) \neq \overline{\alpha}_{\oplus}(v)$ **then** **return** *false*
(3.3') **if** $\overline{\alpha}_{*}(u) \neq \overline{\alpha}_{*}(v)$ **then** **return** *false*
(3.4) **for all** $a \in A$, insert $(\overline{\alpha}_a(u), \overline{\alpha}_a(v))$ into $todo$
(3.5) insert (u, v) into R
(4) **return** *true*

Theorem (Correctness, termination)

Whenever $\mathtt{HKC}^{\infty}(x, y)$ terminates, it returns $\mathtt{true}$ iff $[\![x]\!] = [\![y]\!]$. Moreover, if X is finite then $\mathtt{HKC}^{\infty}(x, y)$ always terminates.


i
a,1
a,1/2
a,1/3
x
a,1/6
y
a,1/3
z
a,1/3
1/3
2/3
1/3
*

$X = \{x, y, z, i\}$

$$\begin{pmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \end{pmatrix} \longleftarrow \begin{bmatrix} x \\ y \\ z \\ i \end{bmatrix}$$

$$\begin{pmatrix} \overline{\alpha}_{\oplus} \\ \overline{\alpha}_{*} \end{pmatrix} \simeq L = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1/3 & 2/3 & 1/3 & 0 \end{pmatrix}$$

$$\overline{\alpha}_a \simeq M_a = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1/6 & 1/3 & 0 & 0 \\ 0 & 0 & 1/3 & 0 \\ 1/2 & 0 & 1/3 & 1 \end{pmatrix}$$

$$\begin{pmatrix}1\\0\\0\\0\end{pmatrix} \xrightarrow{a} \begin{pmatrix}0\\1/6\\0\\1/2\end{pmatrix} \xrightarrow{a} \begin{pmatrix}0\\1/18\\0\\1/2\end{pmatrix} \xrightarrow{\quad a \quad} \cdots \xrightarrow{\ a\ } \begin{pmatrix}0\\1/(2\times 3^n)\\0\\1/2\end{pmatrix} \xrightarrow{\ a\ } \cdots$$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

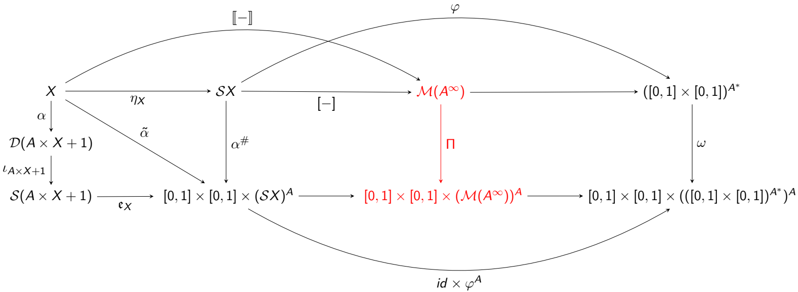
$$\begin{pmatrix}0\\0\\1\\0\end{pmatrix} \xrightarrow{a} \begin{pmatrix}0\\0\\1/3\\1/3\end{pmatrix} \xrightarrow{a} \begin{pmatrix}0\\0\\1/9\\4/9\end{pmatrix} \xrightarrow{\quad a \quad} \cdots \xrightarrow{\ a\ } \begin{pmatrix}0\\0\\1/3^n\\(1-3^{-n})/2\end{pmatrix} \xrightarrow{\ a\ } \cdots$$

Step **Loop**	(3.1) (u, v) extracted from todo	(3.2) Check $(u, v) \in c(R)$	(3.3) Check $Lu = Lv$	(3.4) $(M_a u, M_a v)$ added to todo	(3.5) Cardinality of R
1	$\begin{pmatrix}1\\0\\0\\0\end{pmatrix}, \begin{pmatrix}0\\0\\1\\0\end{pmatrix}$	Fail	$\begin{pmatrix}1\\1/3\end{pmatrix} = \begin{pmatrix}1\\1/3\end{pmatrix}$	$\begin{pmatrix}0\\1/6\\0\\1/2\end{pmatrix}, \begin{pmatrix}0\\0\\1/3\\1/3\end{pmatrix}$	1
2	$\begin{pmatrix}0\\1/6\\0\\1/2\end{pmatrix}, \begin{pmatrix}0\\0\\1/3\\1/3\end{pmatrix}$	Fail	$\begin{pmatrix}2/3\\1/9\end{pmatrix} = \begin{pmatrix}2/3\\1/9\end{pmatrix}$	$\begin{pmatrix}0\\1/18\\0\\1/2\end{pmatrix}, \begin{pmatrix}0\\0\\1/9\\4/9\end{pmatrix}$	2
3	$\begin{pmatrix}0\\1/18\\0\\1/2\end{pmatrix}, \begin{pmatrix}0\\0\\1/9\\4/9\end{pmatrix}$	Success	/	/	2
4	Empty	/	/	/	/

Section 4

Continuous trace semantics

$$\sum \longrightarrow \int$$


⟦−⟧
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M(A∞)
([0, 1] × [0, 1])^A*
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D(A × X + 1)
ι A×X+1
S(A × X + 1)
𝔢X
[0, 1] × [0, 1] × (SX)^A
[0, 1] × [0, 1] × (M(A∞))^A
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id × φ^A

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- **Set** with **Meas** (measurable spaces and measurable functions)

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Replace

- **Set** with **Meas** (measurable spaces and measurable functions)
- $\mathcal{D}$: **Set** $\to$ **Set** with the Giry monad $\mathbb{D}$: **Meas** $\to$ **Meas**

A PTS is a coalgebra $\alpha : X \to \mathbb{D}(A \times X + 1)$ in **Meas**.

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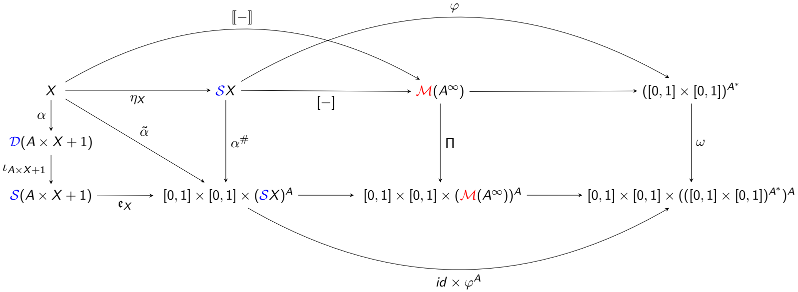
- **Set** with **Meas** (measurable spaces and measurable functions)
- $\mathcal{D}$: **Set** $\to$ **Set** with the Giry monad $\mathbb{D}$: **Meas** $\to$ **Meas**

A PTS is a coalgebra $\alpha : X \to \mathbb{D}(A \times X + 1)$ in **Meas**.

Definition (Trace semantics of α, Kerstan 2013)

By induction on words $w \in A^*$. Here $t_a(x)(S) = \alpha(x)(\{a\} \times S)$.

$$\begin{aligned}
[\![x]\!](\varepsilon) &= \alpha(x)(1) \\
[\![x]\!](\varepsilon A^\infty) &= \alpha(x)(A \times X + 1) \\
[\![x]\!](aw) &= \int_X [\![-]\!](w) dt_a(x) \\
[\![x]\!](awA^\infty) &= \int_X [\![-]\!](wA^\infty) dt_a(x)
\end{aligned}$$


$\llbracket - \rrbracket$
φ
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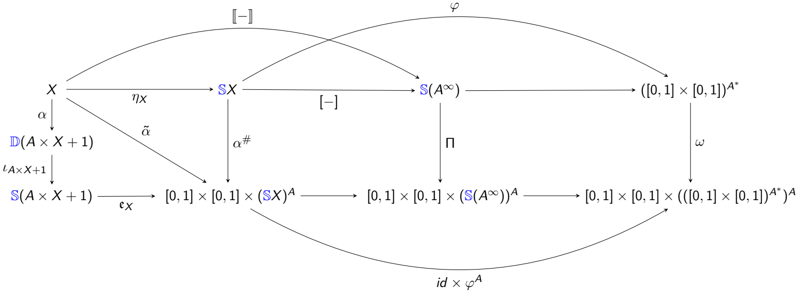
η_X
$\mathcal{S}X$
$[-]$
$\mathcal{M}(A^\infty)$
$([0,1] \times [0,1])^{A^*}$
α
$\tilde{\alpha}$
$\alpha^\#$
Π
ω
$\mathcal{D}(A \times X + 1)$
$\iota_{A \times X + 1}$
$\mathcal{S}(A \times X + 1)$
$\mathfrak{e}_X$
$[0,1] \times [0,1] \times (\mathcal{S}X)^A$
$[0,1] \times [0,1] \times (\mathcal{M}(A^\infty))^A$
$[0,1] \times [0,1] \times (([0,1] \times [0,1])^{A^*})^A$
$id \times \varphi^A$


X
$\mathbb{S}X$
$\mathbb{S}(A^\infty)$
$([0,1] \times [0,1])^{A^*}$
$\mathbb{D}(A \times X + 1)$
$\mathbb{S}(A \times X + 1)$
$[0,1] \times [0,1] \times (\mathbb{S}X)^A$
$[0,1] \times [0,1] \times (\mathbb{S}(A^\infty))^A$
$[0,1] \times [0,1] \times (([0,1] \times [0,1])^{A^*})^A$
$[\![-]\!]$
φ
η_X
$[-]$
α
$\tilde{\alpha}$
$\alpha^\#$
Π
ω
$\iota_{A \times X+1}$
$\mathfrak{e}_X$
$id \times \varphi^A$

Conclusion

- This trace semantics coincides with the known trace semantics, for both discrete and continuous systems.
- This approach is adapted for the discrete case (small amount of measure theory).
- This approach yields an algorithm which computes trace equivalence.

Issues and prospects

- Is it possible to fit into the framework of Jacobs/Silva/Sokolova (2015) ?
- Determinization of a PTS amounts to the passage from a kernel to a stochastic operator

THANK YOU !

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