Boolean-Valued Multiagent Coalgebraic Logic

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(based on work with Alexander Kurz & my Master's thesis)

Fitting's Boolean-Valued logic

"A wide range of many valued logics are in the literature, but one family is notably missing: those whose truth value space is a Boolean algebra other than {false, true}." Fitting (2009)

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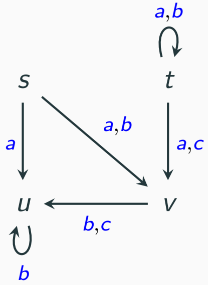
Technically sensible, but conceptually...

Fitting's Boolean-Valued logic

CABA 2^{A} $\sim$ A is a set of agents

Fitting's Boolean-Valued logic

Agent-indexed Kripke models $\mathbb{M} = \langle \mathbb{M}_a \rangle_{a \in A}$



$R : S \times S \to 2^A$

$V : variables \to (2^A)^S$

$(S = states)$

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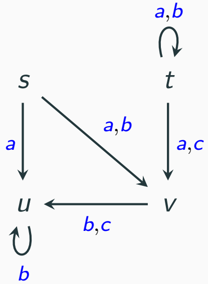

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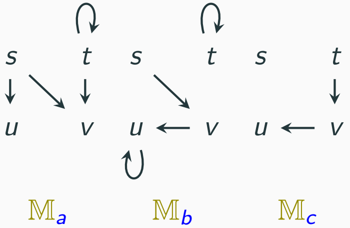

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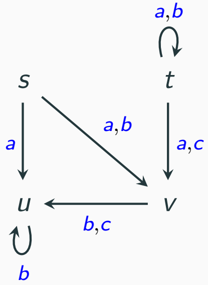

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Formulas same as usual modal logic

Semantics

$$[\![-]\!]_{\mathbb{M}} : \mathit{formulas} \to (2^A)^S$$

satisfying 'Slicing Theorem':

$$[\![\varphi]\!]_{\mathbb{M}}(s) = \{a \in A \,;\, \mathbb{M}_a, s \models \varphi\}$$

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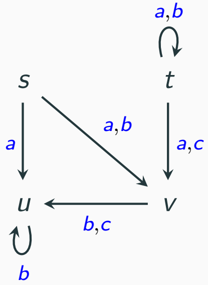

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Truth value of φ is set of agents for whom it is true

Examples Fitting's logic

"Alice and Bob both know that they themselves are wearing hats"

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Assert Kh has truth value $\{a, b\}$

But note: limitations in expressive power (though we can address them)

Fitting's earlier logic

A has a partial (or pre-) ordering of relative expertise

Agent-indexed Kripke models respect the ordering

Truth values are upward closed sets of agents:

$a \in [\![\varphi]\!](s)$ and $a \preceq b$ implies $b \in [\![\varphi]\!](s)$

Interplay!

Questions about Fitting's work

- How do we extend this to other transition structures and modal logics?
- Can we put more structure on the set of agents?
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Coalgebraic logic allows us to tackle these!

Agent-indexed structure

Will keep things simple, no details of enriched category theory, and only the Boolean-valued logic

- Category **ASet** of sets and agent-indexed functions

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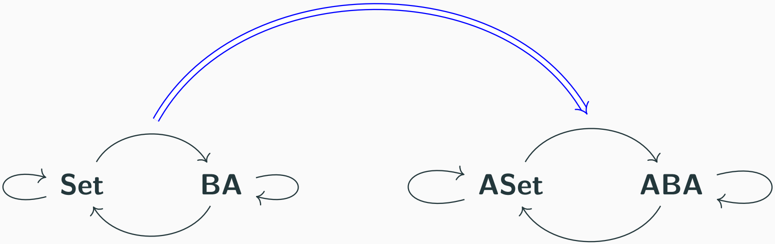
- Category **ABA** of BAs and agent-indexed homomorphisms
- equivalently, Kleisli category of power monad $(-)^A$

Logical connection



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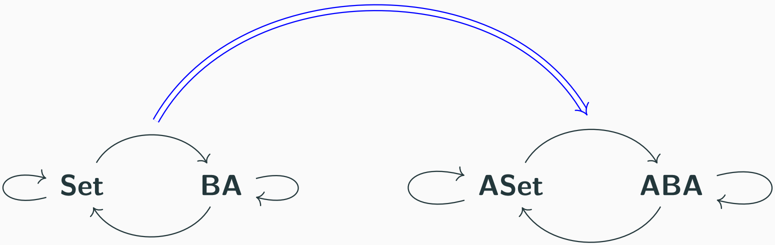

formulas $\to 2^S$

formulas $\to (2^A)^S$

Slicing theorem, adequacy & expressivity, Fitting-style logic given naturally for **ASet**

Wrapping up

Ongoing work:

- All of this works for **Pos** and **DL** (giving expertise order). What other agent structures? Topologies, group actions...
- Strong relation (coalgebraic) Fitting logic to (nondeterministic) multiplayer game semantics and player role distributions fixing expressive power...

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- Fitting-style logics have potential to nicely express situations with agents
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Thank you!

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