Boolean-Valued Multiagent Coalgebraic Logic

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(based on work with Alexander Kurz & my Master’s thesis)
“A wide range of many valued logics are in the literature, but one family is notably **missing:** those whose truth value space is a Boolean algebra other than \{false, true\}.” Fitting (2009)
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Technically sensible, but conceptually...
CABA $2^A$ \sim A$ is a set of agents
Agent-indexed Kripke models $\mathcal{M} = \langle \mathcal{M}_a \rangle_{a \in A}$

$R : S \times S \to 2^A$

$V : \text{variables} \to (2^A)^S$

($S = \text{states}$)
Fitting’s Boolean-Valued logic

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\( S = states \)
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\[ R : S \times S \rightarrow 2^A \]
\[ V : \text{variables} \rightarrow (2^A)^S \]
\( (S = \text{states}) \)

Formulas same as usual modal logic

Semantics

\[ [\phi]_\mathcal{M}(s) = \{ a \in A ; \mathcal{M}_a, s \models \phi \} \]

satisfying ‘Slicing Theorem’:
Fitting’s Boolean-Valued logic

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Semantics

$[\neg]_{\mathcal{M}} : \text{formulas} \rightarrow (2^A)^S$

satisfying ‘Slicing Theorem’:

$\lbrack \varphi \rbrack_{\mathcal{M}}(s) = \{ a \in A ; \mathcal{M}_a, s \models \varphi \}$

Truth value of $\varphi$ is set of agents for whom it is true

$R : S \times S \rightarrow 2^A$

$V : \text{variables} \rightarrow (2^A)^S$

$(S = \text{states})$
Examples Fitting’s logic

“Alice and Bob both know that they themselves are wearing hats”
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‘Usual’ modal logic

Assert $K_a h_a \wedge K_b h_b$ is true
“Alice and Bob both know that they themselves are wearing hats”

‘Usual’ modal logic
Assert $K_a h_a \land K_b h_b$ is true

Fitting’s logic
Assert $Kh$ has truth value $\{a, b\}$
Examples Fitting’s logic

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‘Usual’ modal logic
Assert $K_a h_a \land K_b h_b$ is true

But note: limitations in expressive power (though we can address them)

Fitting’s logic
Assert $Kh$ has truth value $\{a, b\}$
A has a partial (or pre-) ordering of relative expertise

Agent-indexed Kripke models respect the ordering

Truth values are upward closed sets of agents:

\[ a \in \llbracket \varphi \rrbracket(s) \text{ and } a \preceq b \text{ implies } b \in \llbracket \varphi \rrbracket(s) \]

Interplay!
Questions about Fitting’s work

- How do we extend this to other transition structures and modal logics?
- Can we put more structure on the set of agents?
- How do these logics fit in the general picture?
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Coalgebraic logic allows us to tackle these!
Will keep things simple, no details of enriched category theory, and only the Boolean-valued logic

- Category $\mathbf{ASet}$ of sets and agent-indexed functions
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- Category **ASet** of sets and **agent-indexed** functions

- equivalently, **co-Kleisli** category of product/copower comonad $A \times (\cdot)$
Agent-indexed structure

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- Category $\mathbf{ASet}$ of sets and agent-indexed functions
- equivalently, co-Kleisli category of product/copower comonad $A \times (-)$
- lifting $\mathbf{Set}$-functors through dist. laws, $T$-coalgebras in $\mathbf{ASet}$ are agent-indexed $T$-coalgebras in $\mathbf{Set}$
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- Category $\text{ABA}$ of BAs and agent-indexed homomorphisms
- equivalently, Kleisli category of power monad $(-)^A$
Logical connection

Set ← BA → ASet ← ABA → Set

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Logical connection

formulas $\rightarrow 2^S$

formulas $\rightarrow (2^A)^S$

Slicing theorem, adequacy & expressivity, Fitting-style logic given naturally for **ASet**
Wrapping up

Ongoing work:

- All of this works for Pos and DL (giving expertise order). What other agent structures? Topologies, group actions...

- Strong relation (coalgebraic)
  Fitting logic to (nondeterministic) multiplayer game semantics and player role distributions fixing expressive power...

Summing up:

- Fitting-style logics have potential to nicely express situations with agents
- In coalgebraic generalisation, Fitting-style logics are naturally associated to agent-indexed structures

Thank you!
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